

4 METHODS of SOLVING QUADRATIC EQUATIONS

1. What makes an equation a quadratic equation? Highest power is 2
2. There are four methods. List them!
 1. Square roots method
 2. Factor & Zero Product property
 3. Complete the square
 4. Quadratic Formula
3. How can you determine which method to use?
 - a. If the equation has x^2 (no x'),
Then use the square roots method.
 - b. If the equation has x^2 and x and the trinomial is factorable
Then factor it and use the zero product property
 - c. If the equation has x^2 and x and the trinomial is not factorable
Then use the complete the square method
 - d. If completing the square fails (the Bx-term is odd)
Then use the Quadratic Formula!

WHICH METHOD WOULD YOU USE? SQ, ZPP, C, QF

- | | | | |
|------------------------|------------|-----------------------|------------|
| 1. $x^2 - 6x + 5 = 0$ | <u>ZPP</u> | 2. $4x^2 + 100 = 14$ | <u>SQ</u> |
| 3. $9x^2 - 100 = 0$ | <u>SQ</u> | 4. $x^2 + 8x + 3 = 0$ | <u>CTS</u> |
| 5. $-2 = 2x^2 + 8$ | <u>SQ</u> | 6. $x^2 + 3x + 1 = 0$ | <u>QF</u> |
| 7. $4x^2 + 4x + 1 = 0$ | <u>ZPP</u> | 8. $x^2 + 6x + 3 = 0$ | <u>C</u> |

SOLVING: WHICH METHOD SHOULD YOU USE?

Explain why!

	Equation	A	B	C	D
1	$x^2 + 4x + 3 = 0$ $(x+3)(x+1) = 0$	Sq. Roots $x^2 \neq x$	Factor/ZPP It factors!	Complete Sq.	Quad. Form
2	$5x^2 - 1 = 6$	Sq. Roots x^2	Factor/ZPP	Complete Sq.	Quad. Form
3	$x^2 - 7x + 1 = 0$	Sq. Roots $x^2 \neq x$	Factor/ZPP Does not factor	Complete Sq. B is odd	Quad. Form
4	$x^2 + 10x + 4 = 0$	Sq. Roots $x^2 \neq x$	Factor/ZPP Does not factor	Complete Sq. B is even	Quad. Form
5	$x^2 - 14x = 5$	Sq. Roots $x^2 \neq x$	Factor/ZPP Does not factor	Complete Sq. B is even	Quad. Form
6	$5 - 3x^2 = 20$	Sq. Roots x^2	Factor/ZPP	Complete Sq.	Quad. Form
7	$x^2 + x = 10$	Sq. Roots $x^2 \neq x$	Factor/ZPP Does not factor	Complete Sq. B is odd	Quad. Form
8	$x^2 - 4x - 12 = 0$ $(x-6)(x+2) = 0$	Sq. Roots $x^2 \neq x$	Factor/ZPP It factors!	Complete Sq.	Quad. Form

NAME: _____

Solve each equation (use the method provided)

1. Square Roots Method

$$(x+3)^2 + 2 = -10$$

-2 -2

$$\sqrt{(x+3)^2} = \sqrt{-12}$$

$$x+3 = \pm \sqrt{-4} \sqrt{3}$$

$$x+3 = \pm 2i\sqrt{3}$$

-3 -3

$$\boxed{x = -3 \pm 2i\sqrt{3}}$$

2. Factor to solve.

$$x^2 - 2x - 15 = 0$$

$$(x+3)(x-5) = 0$$

$\begin{array}{r} 15 \\ 1 \ 15 \\ +3 -5 \end{array}$

$$x+3=0$$

-3 -3

$$x-5=0$$

+5 +5

$$x = -3$$

$$x = 5$$

$$\boxed{x = -3, 5}$$

3. Complete the Square.

$$x^2 - 8x + 3 = 0$$

-3 -3

$$x^2 - 8x = -3$$

$$x^2 - 8x + 16 = -3 + 16$$

$$\left(\frac{-8}{2}\right)^2 = 16$$

$$\sqrt{(x-4)^2} = \sqrt{13}$$

$$x-4 = \pm \sqrt{13}$$

+4 +4

$$\boxed{x = 4 \pm \sqrt{13}}$$

4. Quadratic Formula.

$$6x^2 + 2x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(6)(1)}}{2(6)}$$

$$x = \frac{-2 \pm \sqrt{4 - 24}}{12}$$

$$x = \frac{-2 \pm \sqrt{-20}}{12}$$

$$x = \frac{-2 \pm \sqrt{-4\sqrt{5}}}{12}$$

$$x = \frac{-2 \pm 2i\sqrt{5}}{12}$$

$$\boxed{x = \frac{-1 \pm i\sqrt{5}}{6}}$$

Solving Quadratic Equations

Solve each quadratic equation (use a different method for each question).

Use the Quadratic Formula on (#1) only.

5. $2x^2 + 5x + 3 = 0$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

$$x = \frac{-5 \pm \sqrt{1}}{4}$$

$$x = \frac{-5 \pm 1}{4}$$

$$x = \frac{-5+1}{4}, \frac{-5-1}{4} = \frac{-6}{4}$$

$$x = -1, -\frac{3}{2}$$

6. $x^2 + 5x + 4 = 0$

$$(x+4)(x+1) = 0$$

$$x+4 = 0$$

$$x+1 = 0$$

$$x = -4$$

$$x = -1$$

$$x = -4, -1$$

7. $-2 + 5x^2 = -62$

$$\begin{array}{r} +2 \quad +2 \\ \hline \end{array}$$

$$\frac{5x^2}{5} = \frac{-60}{5}$$

$$\sqrt{x^2} = \sqrt{-12}$$

$$x = \pm \sqrt{-4} \sqrt{3}$$

$$x = \pm 2i\sqrt{3}$$

8. $x^2 + 14x + 1 = 0$

$$\begin{array}{r} -1 \quad -1 \\ \hline \end{array}$$

$$x^2 + 14x = -1 \quad \left(\frac{14}{2}\right)^2 = 49$$

$$x^2 + 14x + 49 = -1 + 49$$

$$\sqrt{(x+7)^2} = \sqrt{48}$$

$$x+7 = \pm \sqrt{16} \sqrt{3}$$

$$x+7 = \pm 4\sqrt{3}$$

$$x = -7 \pm 4\sqrt{3}$$

SOLVING QUADRATIC EQUATIONS (Completing the Square A=1)

Step 1: Arrange the equation in the form $Ax^2 + Bx = C$

$$\begin{array}{r} A \quad B \quad C \\ 1x^2 + 10x - 3 = 0 \\ \quad \quad +3 \quad +3 \\ \hline \end{array}$$

Step 2: Determine what value for the third term will make the trinomial a perfect square.

$$\left(\frac{B}{2}\right)^2 = C \quad \left(\frac{10}{2}\right)^2 = 5^2 = 25$$

$$\begin{array}{r} x^2 + 10x = 3 \\ \quad \quad +25 \quad +25 \end{array}$$

Step 3: Add that value to both sides.

$$x^2 + 10x + 25 = 28$$

Step 4: Simplify (write the trinomial as a binomial squared).

$$\sqrt{(x+5)^2} = \sqrt{28}$$

Step 5: Take the square root of both sides. (remember the \pm)

$$\begin{array}{l} \sqrt{28} \\ \sqrt{4}\sqrt{7} \\ 2\sqrt{7} \end{array}$$

$$\begin{array}{l} x+5 = \pm \sqrt{28} \\ x+5 = \pm 2\sqrt{7} \\ \quad -5 \quad -5 \end{array}$$

Step 6: Solve for x (often there are 2 solutions)

$$\boxed{x = -5 \pm 2\sqrt{7}}$$

2. Solve by completing the square.

$$\begin{array}{r} x^2 - 4x = 2x + 35 \\ \quad -2x \quad -2x \\ \hline \end{array}$$

$$\begin{array}{r} x^2 - 6x = 35 \\ \quad +9 \quad +9 \end{array}$$

$$x^2 - 6x + 9 = 44$$

$$\sqrt{(x-3)^2} = \sqrt{44}$$

$$x-3 = \pm \sqrt{44}$$

$$x-3 = \pm \sqrt{4}\sqrt{11}$$

$$\begin{array}{r} x-3 = \pm 2\sqrt{11} \\ \quad +3 \quad +3 \end{array}$$

$$\boxed{x = 3 \pm 2\sqrt{11}}$$

$$\left(\frac{B}{2}\right)^2 = C$$

$$\left(\frac{-6}{2}\right)^2 = C$$

$$(-3)^2 = C$$

$$9 = C$$

SOLVING QUADRATIC EQUATIONS (Completing the Square $A \neq 1$)

Step 1: Arrange the equation in the form $Ax^2 + Bx = C$

$$\frac{3x^2 - 6x + 12 = 0}{3 \quad 3}$$

Step 2: Divide both sides by "A".

$$x^2 - 2x + 4 = 0$$

Step 3: Determine what value for the third term will make the trinomial a perfect square. $\left(\frac{-2}{2}\right)^2 = 1$
Then Add that value to both sides.

$$x^2 - 2x = -4$$

Step 5: Simplify (write the trinomial as a binomial squared).

$$x^2 - 2x + 1 = -3$$

Step 6: Take the square root of both sides.
(remember the \pm)

$$\sqrt{(x-1)^2} = \sqrt{-3}$$

Step 7: Solve for x (often there are 2 solutions)

$$x - 1 = \pm i\sqrt{3}$$

$$x = 1 \pm i\sqrt{3}$$

Solve All Types of Equations

Example 1: Polynomial: $x^3 = x^2$

$$\begin{array}{r} x^3 - x^2 = 0 \\ \underline{-x^2 - x^2} \\ x^3 - x^2 = 0 \\ x^2(x-1) = 0 \end{array}$$

What could x be? 0 or 1

$$\begin{array}{l|l} x^2 = 0 & x-1 = 0 \\ \hline \boxed{x=0} & \boxed{x=1} \end{array}$$

<p>1. $2-x = -1$</p> <p>Abs Val cannot be equal to a negative.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">No Solution</div>	<p>2. $x^2 + \sqrt{2}x - 2 = 0$</p> <p style="text-align: right;">$a=1$ $b=\sqrt{2}$ $c=-2$</p> $x = \frac{-\sqrt{2} \pm \sqrt{\sqrt{2}^2 - 4(1)(-2)}}{2(1)}$ $x = \frac{-\sqrt{2} \pm \sqrt{2+8}}{2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">$x = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$</div>
<p>3. $\sqrt[3]{3t+1} = (-2)^3$</p> $\frac{3t+1}{-1} = \frac{-8}{-1}$ $\frac{3t}{3} = \frac{-9}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">$t = -3$</div> <div style="margin-left: 20px;"> <p>Check</p> $\sqrt[3]{3(-3)+1} = -2$ $\sqrt[3]{-9+1} = -2$ $\sqrt[3]{-8} = -2$ <p style="text-align: center;">✓</p> </div>	<p>4. $\sqrt{12-x} = x^2$</p> $\frac{12-x}{-12+x} = \frac{x^2}{x-12}$ $0 = x^2 + x - 12$ $0 = (x+4)(x-3)$ $x = -4, 3$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">$x = 3$</div> <div style="margin-left: 20px;"> <p>Check</p> $\sqrt{12-(-4)} = -4$ $\sqrt{16} = -4$ $4 \neq -4$ <p>Check</p> $\sqrt{12-3} = 3$ $\sqrt{9} = 3$ <p style="text-align: center;">✓</p> </div>
<p>5. $\frac{-2}{x+4} = \frac{-3}{x+1}$</p> $\frac{-2(x+1)}{-2x-2} = \frac{-3(x+4)}{-3x-12}$ $\frac{-2x-2}{-2x-2} = \frac{-3x-12}{-3x-12}$ $\frac{-2x-2}{-2x-2} = \frac{-3x-12}{-3x-12}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">$x = -10$</div>	<p>6. $\left(\frac{3x}{x+1} + \frac{6}{2x} = \frac{7}{x}\right) \times \text{LCD} = 2x(x+1)$</p> $\frac{3x(2x)(x+1)}{(x+1)} + \frac{6(2x)(x+1)}{2x} = \frac{7(2x)(x+1)}{x}$ $6x^2 + 6(x+1) = 14(x+1)$ $6x^2 + 6x + 6 = 14x + 14$ $6x^2 - 8x - 8 = 0$ $3x^2 - 4x - 4 = 0$ $(3x+2)(x-2) = 0$

$$\boxed{x = -\frac{2}{3}, 2} \text{ check } \checkmark$$