Functions with e and the Natural Log

DAY 3

$\left(1+\frac{1}{1}\right)^{1}$	
$\left(1+\frac{1}{2}\right)^{2}$	
$\left(1+\frac{1}{10}\right)^{10}$	
$\left(1+\frac{1}{50}\right)^{50}$	
$\left(1+\frac{1}{100}\right)^{100}$	
$\left(1+\frac{1}{\_\_\_\_}\right)$	
$\left(1+\frac{1}{\_\_\_\_}\right)$	
$\left(1+\frac{1}{\_\_\_\_}\right)$	

The history of mathematics is marked by the discovery of special number such as π and i. Another such number is the natural base, e.

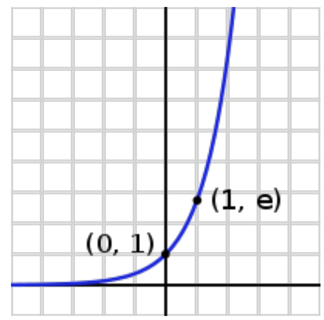
e: The expression $\left(1+\frac{1}{n}\right)^n$ approaches e as n approaches infinity.

The number e is defined as the number that $\left(1+\frac{1}{n}\right)^n$ approaches as n becomes infinitely large.

Note: $e \approx 2.718$ when graphing with 'e', treat it as a little less than '3'

The important mathematical constant e, sometimes called Euler's number, is approximately equal to 2.718 and is the base of the natural logarithm.

$$e = \lim_{n \to \infty}\left(1+\frac{1}{n}\right)^n$$



Transformations of Exponential Graphs

Function	Movement
$f(x) = e^x$	
$f(x) = e^{-x}$	
$f(x) = -e^x$	
$f(x) = e^{x+a}$	
$f(x) = e^{x-a}$	
$f(x) = e^x - b$	
$f(x) = e^x + b$	

1. Explain how the graphs below would move given the parent graph of $f(x)=e^x$

a) $f(x)=e^{x+2}$

b) $f(x)=e^x-1$

c) $f(x)=5+e^x$

d) $f(x)=9-e^{-x}$

Example 2 → Simplifying the Natural Base (all the same rules)

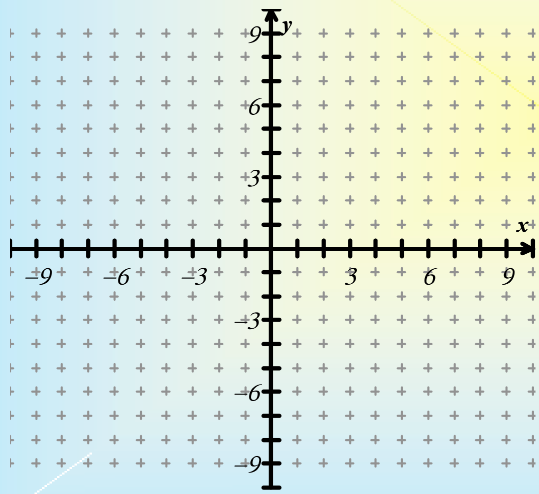
a) $e^7 \bullet e^2$ b) $e^{-12} \bullet e^7$ c) $\left(e^8\right)^5$

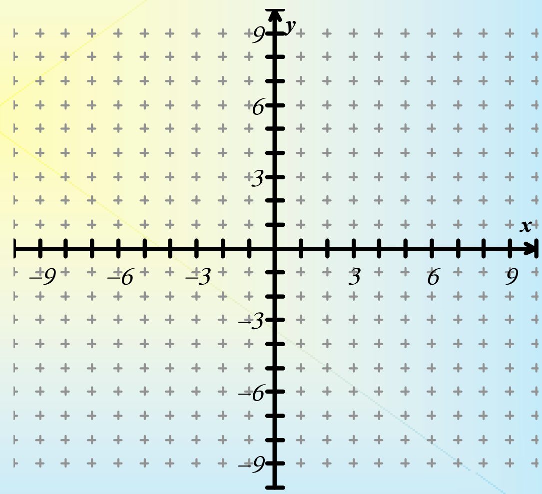
Natural Base Functions

The function $y = a\,e^{rx}$ is called a natural base function.

- If $a > 0$ and $r > 0$, the function is an exponential growth function
- If $a > 0$ and $r < 0$, the function is an exponential decay function

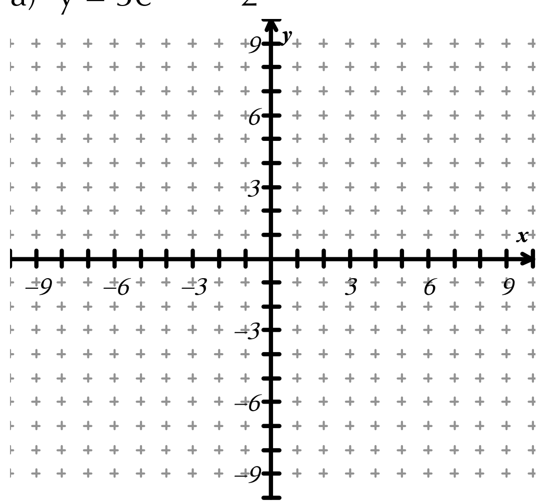
Graph the function below!



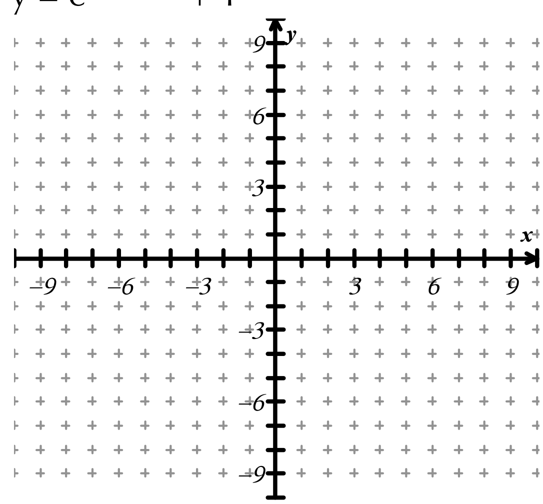


Example 3 → Graph the natural base functions

a) $y = 3e^{0.25x} - 2$



b) $y = e^{-0.75(x-2)} + 1$



Continually Compounded Interest

If the frequency of compounding approached infinity, the compound interest formula can change to be…

$$A = Pe^{rt}$$

Example 4→ Modeling compound interest

You deposit $4,000 in a bank account that earns 6% annual interest. How much money will you have in the account after 10 years if interest is compounded…

a) Annually?

b) Monthly?

c) Continually?

Natural Log

Logarithm with a base of *e*.

The natural log is denotes ln(x).

If $f(x) = e^x$ and $g(x) = \ln(x)$, then f and g are inverse functions

EXAMPLE 5→ Simplify the expression

$\ln(e^{3x-4})$

$e^{\ln(2x)}$

EXAMPLE 6→ Find the inverse function

$y = \frac{1}{4}e^{x-2}$

$y = -2\ln(x) - 4$