Ch 7.3 Law of Cosines, Area of a Triangle DAY 3

Law of Cosines

- In any ΔABC, with sides a, b, and c,
  - $a^2 = b^2 + c^2 - 2bc\cos\alpha$
  - $b^2 = a^2 + c^2 - 2ac\cos\beta$
  - $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Triangle Side Length Restriction

- In any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.

1. Use the Law of Cosines to solve the triangle.
   $\alpha = 42.3°,\ b = 12.9m,\ c = 15.4m$.

2. Use the Law of Cosines to solve the triangle.
   $\gamma = 72.40°,\ a = 327\text{ft},\ \text{and}\ b = 251\text{ft}.$

3. Use the Law of Cosines to solve the triangle.
$a = 9.47\text{ft},\quad b = 15.9\text{ft}\quad \text{and}\quad c = 21.1\text{ft}$.

Area of any triangle -
equals one-half the product of the length of two sides times the sine of their **included** angle.

$$\mathbf{A_\Delta} = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$$

Find the area of the triangle having the indicated sides and angle.

4. β = 72°, a = 105, and c = 64

5. a = 12, $\alpha = 30°$, and $\beta = 61°$.

Heron's Area Formula

If a triangle has sides of length a, b, and c, and if the semiperimeter is $s = \frac{1}{2}(a+b+c)$,

then the area of the triangle is: $A = \sqrt{s(s-a)(s-b)(s-c)}$.

6. Find the area of the triangle having sides of lengths
$a = 29.7\,ft,\ b = 42.3\,ft,\ \text{and}\ c = 38.4\,ft.$