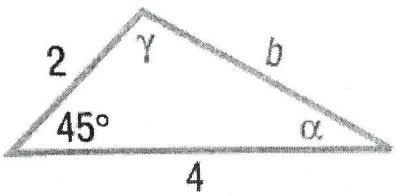
HOMEWORK: LAW OF COSINES

NAME: ______________________ DAY 3 DUE: __________

Solve each triangle.

1.



$b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = 2^2 + 4^2 - 2(2)(4)\cos 45^\circ$

$b^2 = 20 - 16\left(\frac{\sqrt{2}}{2}\right)$

$b^2 = 2 - 8\sqrt{2}$

$\boxed{b \approx 2.95}$

$a^2 = b^2 + c^2 - 2bc(\cos\alpha)$

$\frac{2bc(\cos\alpha)}{2bc} = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos\alpha = \frac{2.95^2 + 4^2 - 2^2}{2(2.95)(4)}$

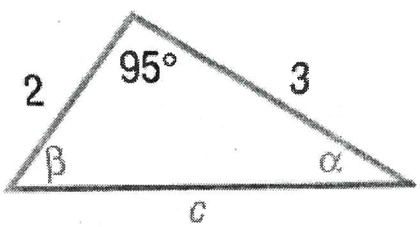
$\cos\alpha = \frac{20.703}{23.6}$

$\boxed{\alpha \approx 28.7}$

$\gamma = 180 - 28.7 - 45$

$\boxed{\gamma = 106.3^\circ}$

2.



$c^2 = a^2 + b^2 - 2ab(\cos\gamma)$

$c^2 = 2^2 + 3^2 - 2(2)(3)\cos 95^\circ$

$c^2 = 13 - 12\cos 95^\circ$

$\boxed{c \approx 3.75}$

$a^2 = b^2 + c^2 - 2bc(\cos\alpha)$

$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos\alpha = \frac{3^2 + 3.75^2 - 2^2}{(2)(3)(3.75)}$

$\cos\alpha = \frac{19.063}{22.5}$

$\boxed{\alpha \approx 32.1^\circ}$

$\beta = 180^\circ - 32.1^\circ - 95^\circ$

$\boxed{\beta \approx 52.9^\circ}$

3. $a = 3,\ b = 4,\ \gamma = 40°$.

$c^2 = a^2 + b^2 + 2ab(\cos\gamma)$

$c^2 = 3^2 + 4^2 + 2(3)(4)\cos 40°$

$\boxed{c \approx 2.57}$

$a^2 = b^2 + c^2 + 2bc(\cos\alpha)$

$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\alpha = \dfrac{4^2 + 2.57^2 - 3^2}{2(4)(2.57)}$

$\cos\alpha = \dfrac{13.605}{20.56}$

$\boxed{\alpha \approx 48.6°}$

$\beta = 180° - 48.6° - 40°$

$\boxed{\beta \approx 91.4°}$

4. $b = 1,\ c = 3,\ \alpha = 80°$.

$a^2 = b^2 + c^2 + 2bc(\cos\alpha)$

$a^2 = 1^2 + 3^2 + 2(1)(3)(\cos 80°)$

$a^2 = 10 - 6(\cos 80°)$

$a \approx 2.99$

$b^2 = a^2 + c^2 + 2ac(\cos\beta)$

$\cos\beta = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos\beta = \dfrac{2.99^2 + 3^2 - 1^2}{2(2.99)(3)}$

$\cos\beta = \dfrac{16.940}{17.94}$

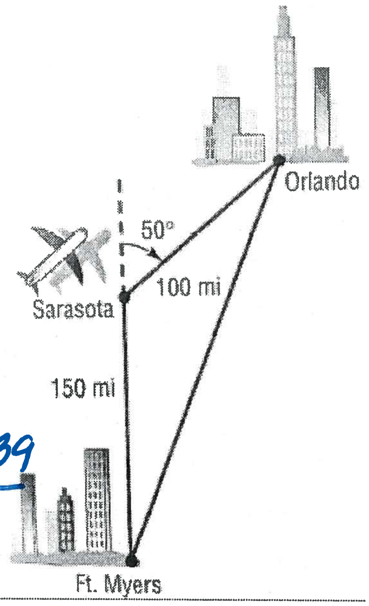
$\boxed{\beta \approx 19.2°}$

$\gamma = 180° - 19.2° - 80°$

$\boxed{\gamma \approx 80.8°}$

5. A cruise ship maintains an average speed of 15 knots in going from San Juan, Puerto Rico, to Barbados, West Indies, a distance of 600 nautical miles. To avoid a tropical storm, the captain heads out of San Juan in a direction of 20° off a direct heading to Barbados. The captain maintains the 15-knot speed for 10 hours, after which time the path to Barbados becomes clear of storms.

a) Through what angle should the captain turn to head directly to Barbados?



$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 600^2 + 150^2 - 2(600)(150)(\cos 20°)$

$c^2 = 382{,}500 - 180{,}000\cos 20°$

$\boxed{c \approx 461.9 \text{ nautical miles}}$

$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\alpha = \dfrac{150^2 + 461.9^2 - 600^2}{2(150)(461.9)} = \dfrac{-124{,}148.39}{138{,}570}$

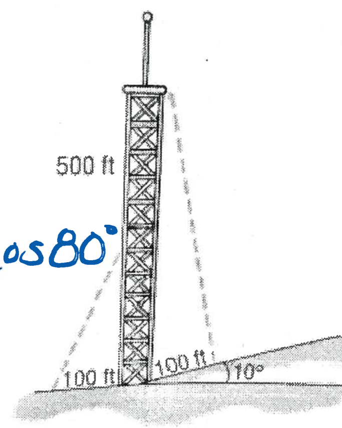
$\boxed{\alpha \approx 153.6°}$ $\quad x = 180 - 153.6 = \boxed{26.4°}$

b) Once the turn is made, how long will it be before the ship reaches Barbados if the same 15-knot speed is maintained?

$t = \dfrac{461.9}{15} \approx \boxed{30.8 \text{ hours}}$

6. The height of a radio tower is 500 feet, and the ground on one side of the tower slopes upward at an angle of 10°.

a) How long should a guy wire be if it is to connect to the top of the tower and be secured at a point on the sloped side 100 feet from the base of the tower?



$x^2 = 500^2 + 100^2 - 2(500)(100)\cos 80°$

$x^2 = 260{,}000 - 100{,}000\cos 80°$

$\boxed{x = 492.6 \text{ ft}}$

b) How long should a second guy wire be if it is to connect to the middle of the tower and be secured at a point 100 feet from the base on the flat side?

$100^2 + 250^2 = y^2$

$72{,}500 = y^2$

$\boxed{y \approx 269.3 \text{ ft}}$

7. Find the area: a = 6, b = 5, c = 8

$s = \frac{1}{2}(a+b+c)$

$s = \frac{1}{2}(6+5+8) \rightarrow s = \frac{19}{2}$

$A = \sqrt{\frac{19}{2}\left(\frac{19}{2}-\frac{12}{2}\right)\left(\frac{19}{2}-\frac{10}{2}\right)\left(\frac{19}{2}-\frac{16}{2}\right)}$

$A = \sqrt{\frac{19}{2}\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)\left(\frac{3}{2}\right)}$

$A = \sqrt{\frac{3591}{16}} = \boxed{14.98}$

8. Find the area: a = 2, b = 2, c = 2

$s = \frac{1}{2}(2+2+2)$

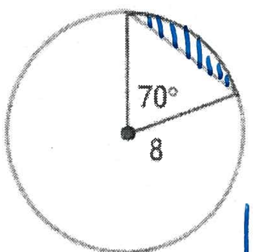
$s = 3$

$A = \sqrt{3(3-2)(3-2)(3-2)}$

$A = \sqrt{3(1)}$

$A = \sqrt{3} = \boxed{1.73}$

9. Find the area of the shaded region.



Area of Sector

$\theta = 70°\left(\frac{\pi}{180}\right) = \frac{7\pi}{18}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(8)^2\left(\frac{7\pi}{18}\right)$

$A = \frac{112\pi}{9}\ ft^2$

Area of Triangle

$A = \frac{1}{2}r \cdot r\sin\theta$

$A = \frac{1}{2}(8) \cdot 8\sin 70°$

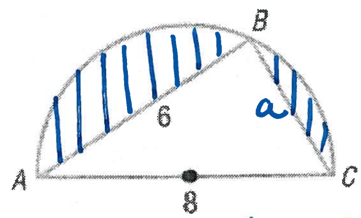
$A = 32\sin 70°$

AREA

$\frac{112\pi}{9} - 32\sin 70°$

$\boxed{A \approx 9.03 ft^2}$

10. Find the area of the shaded region.



$a^2+b^2=c^2$

$a^2+6^2=8^2$

$a^2+36=64$

$a^2=28$

$a=\sqrt{28}$

$a=2\sqrt{7}$

Semicircle Area

$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(4)^2$

$A = 8\pi\ cm^2$

Triangle Area

$A = \frac{1}{2}(6)(2\sqrt{7})$

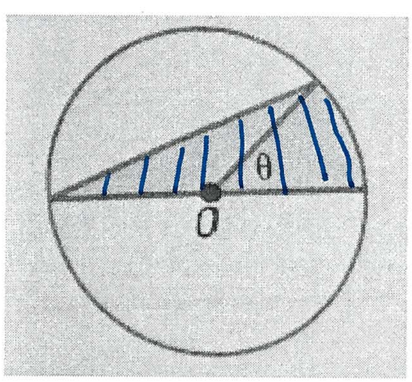
$A = 6\sqrt{7}$

AREA

$A = 8\pi - 6\sqrt{7}$

$\boxed{A \approx 9.26\ cm^2}$

11. Find the area of the shaded region as a function of the central angle θ.



Area of Triangle

$A = \frac{1}{2}r \cdot r\sin(\pi-\theta)$

$A = \frac{1}{2}r^2\sin(\pi-\theta)$

Area of Sector

$A = \frac{1}{2}r^2\theta$

$AREA = \frac{1}{2}r^2\sin(\pi-\theta) + \frac{1}{2}r^2\theta$

$AREA = \frac{1}{2}r^2(\sin(\pi-\theta) + \theta)$

$AREA = \frac{1}{2}r^2(\sin\pi\cos\theta - \cos\pi\sin\theta + \theta)$

$AREA = \frac{1}{2}r^2(0 - \sin\theta + \theta)$

$\boxed{AREA = \frac{1}{2}r^2(\sin\theta + \theta)}$