Ch 7.3 Law of Cosines, Area of a Triangle DAY 3

✓

Law of Cosines

- In any ΔABC, with sides a, b, and c,
  - $a^2 = b^2 + c^2 - 2bc\cos\alpha$
  - $b^2 = a^2 + c^2 - 2ac\cos\beta$
  - $c^2 = a^2 + b^2 - 2ab\cos\gamma$

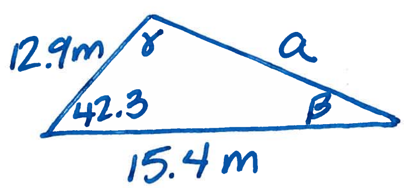
SAS

SSS

Triangle Side Length Restriction

- In any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.

1. Use the Law of Cosines to solve the triangle.
   $\alpha = 42.3°$, $b = 12.9m$, $c = 15.4m$.



$a^2 = (1.29)^2 + (15.4)^2 - 2(12.9)(15.4)\cos 42.3$

$a^2 = 109.69977\ldots$

$\boxed{a \approx 10.474\,m}$

$\dfrac{\sin 42.3}{10.474} = \dfrac{\sin\beta}{12.9}$

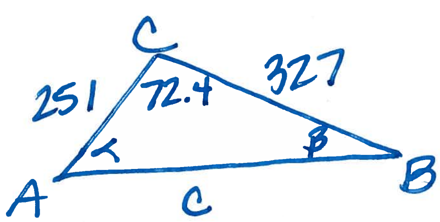
$\sin\beta = \dfrac{12.9\sin 42.3}{10.474}$

$\boxed{\beta \approx 55.986°}$

$\gamma = 180° - 42.3° - 55.986°$

$\boxed{\gamma = 81.714°}$

2. Use the Law of Cosines to solve the triangle.
   $\gamma = 72.40°$, $a = 327\text{ft}$, and $b = 251\text{ft}$.



$c^2 = (327)^2 + (251)^2 - 2(327)(251)\cos 72.4$

$c^2 = 120{,}294.517$

$\boxed{c \approx 346.835}$

$\dfrac{\sin 72.4}{346.835} = \dfrac{\sin\alpha}{327}$

$\sin\alpha = \dfrac{327\sin 72.4}{346.835}$

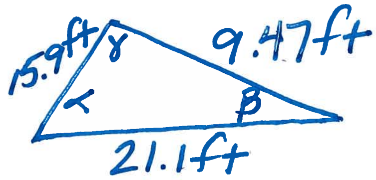
$\alpha = \sin^{-1}\left(\dfrac{327\sin 72.4}{346.835}\right)$

$\boxed{\alpha \approx 63.985°}$

$\beta = 180° - 72.4° - 63.985°$

$\boxed{\beta \approx 43.615°}$

3. Use the Law of Cosines to solve the triangle.
$a = 9.47\text{ft}, \quad b = 15.9\text{ft} \quad \text{and} \quad c = 21.1\text{ft}$.



$9.47^2 = 15.9^2 + 21.1^2 - 2(15.9)(21.1)\cos\alpha$

$\dfrac{9.47^2 - 15.9^2 - 21.1^2}{-670.98} = \dfrac{-670.98\cos\alpha}{-670.98}$

$\dfrac{-608.34}{-670.98} = \cos\alpha$

$\boxed{\alpha \approx 24.955^\circ}$

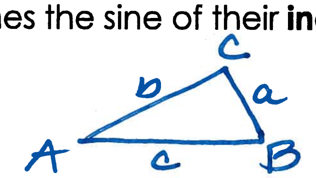
$\dfrac{\sin 24.955}{9.47} = \dfrac{\sin\beta}{15.9}$

$\sin\beta = \dfrac{15.9\sin 24.955}{9.47}$

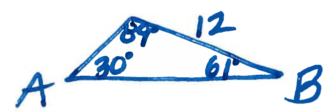
$\boxed{\beta \approx 45.103^\circ}$, $\boxed{\gamma = 109.942^\circ}$

Area of any triangle - SSS
equals one-half the product of the length of two sides times the sine of their **included** angle.

$$\mathbf{A}_\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$$



Find the area of the triangle having the indicated sides and angle.



4. $\beta = 72°$, a = 105, and c = 64

$A = \frac{1}{2}(105)(64)\sin 72^\circ$

$A = 3360\sin 72^\circ$

$\boxed{A \approx 3195.55}$

5. a = 12, $\alpha = 30°$, and $\beta = 61°$.

$\dfrac{\sin 30}{12} = \dfrac{\sin 89}{c}$

$\dfrac{c\sin 30^\circ}{\sin 30^\circ} = \dfrac{12\sin 89^\circ}{\sin 30^\circ}$

$\boxed{c \approx 23.996}$

$A = \frac{1}{2}(12)(23.996)\sin 61^\circ$

$A = (143.978)\sin 61^\circ$

$\boxed{A \approx 125.926}$

<u>Heron's Area Formula</u>

If a triangle has sides of length a, b, and c, and if the semiperimeter is $s = \frac{1}{2}(a+b+c)$,

then the area of the triangle is: $A = \sqrt{s(s-a)(s-b)(s-c)}$.

6. Find the area of the triangle having sides of lengths
$a = 29.7\,ft, \quad b = 42.3\,ft, \quad \text{and} \quad c = 38.4\,ft$.

$S = \frac{1}{2}(29.7 + 42.3 + 38.4)$

$S = 55.2$

$A = \sqrt{55.2(55.2-29.7)(55.2-42.3)(55.2-38.4)}$

$\boxed{A \approx 552.318\text{ ft}^2}$