

# Ch 9.3 – Ellipses

DAY 4

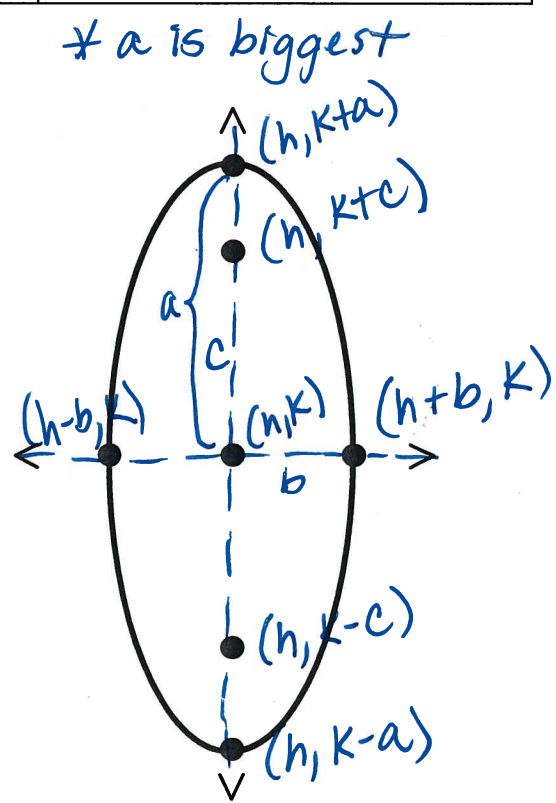
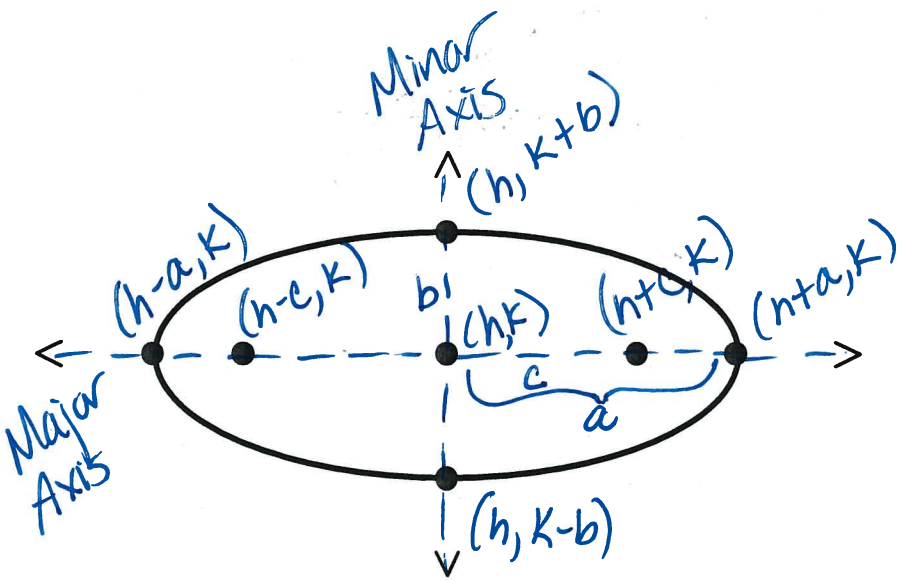
**Ellipse:** the **set of points** in a plane, the sum of whose distances from two fixed points, called **foci**, is a constant.

Important components of the ellipse:

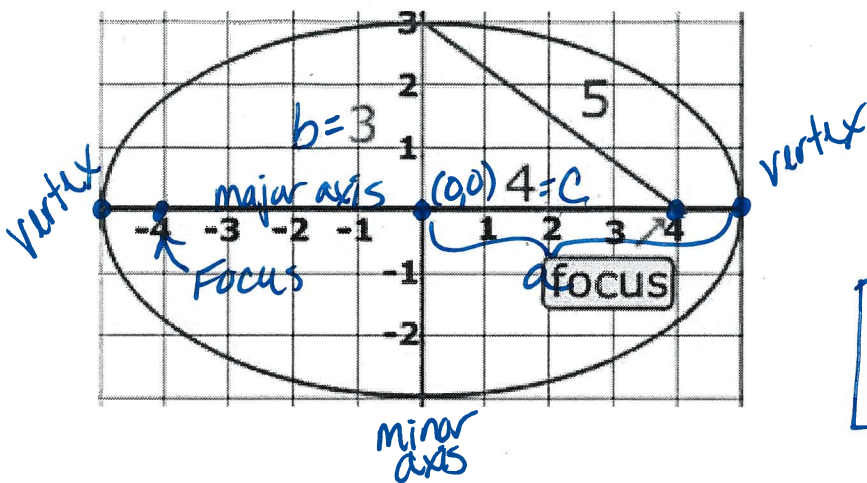
- The line containing the foci is the **major axis**.
- The midpoint of the segment containing the foci is the **center** of the ellipse.
- The line through the center, perpendicular to the major axis, is the **minor axis**.
- The two points of intersection of the ellipse and the major axis are the **vertices** of the ellipse.

**Equation of an ellipse with center at the origin:**

Major Axis	Foci	Vertices	Equation
x-axis	$(-c, 0)$ and $(c, 0)$	$(-a, 0)$ and $(a, 0)$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a > b > 0$ and $b^2 = a^2 - c^2$
y-axis	$(0, -c)$ and $(0, c)$	$(0, -a)$ and $(0, a)$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ $a > b > 0$ and $b^2 = a^2 - c^2$



Identify the important components....



Vertex:  $(-5,0), (5,0)$

Foci:  $(-4,0), (4,0)$

$b=3, c=4,$

$$b^2 = a^2 - c^2$$

$$3^2 = a^2 - 4^2$$

$$9 = a^2 - 16$$

$$25 = a^2$$

$$5 = a$$

$$\frac{(x-0)^2}{25} + \frac{(y-0)^2}{9} = 1$$

Example 1: Graph  $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$

Center:  $(3, -1)$

$a=3$        $b^2 = a^2 - c^2$

$b=2$        $4 = 9 - c^2$

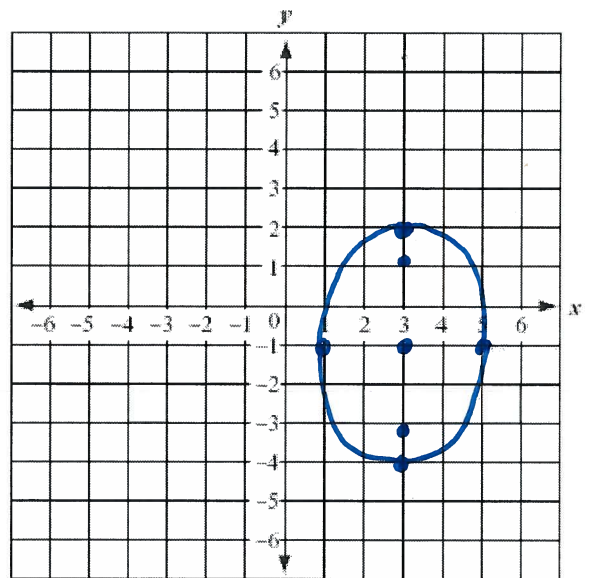
$c = \sqrt{5}$        $+5 = +c^2$

$c = \sqrt{5}$

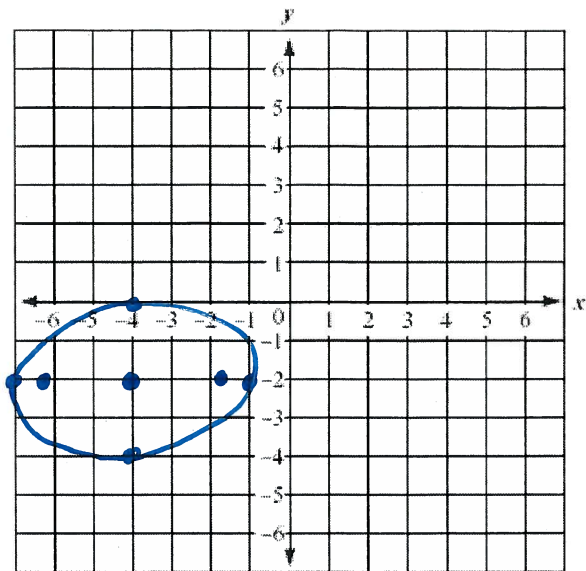
Vertices:  $(3, 2), (3, -4)$

side vertices:  $(1, -1), (5, -1)$

Foci:  $(3, -1 + \sqrt{5}), (3, -1 - \sqrt{5})$



Example 2: Graph  $\frac{(x+4)^2}{9} + \frac{(y+2)^2}{4} = 1$



Center:  $(-4, -2)$

$a = 3$

$b = 2$

$c = \sqrt{5}$

vertices:  $(-7, -2)$  side:  $(-4, 0)$   
 $(-1, -2)$   $(-4, -4)$

foci:  $(-4 - \sqrt{5}, -2), (-4 + \sqrt{5}, -2)$

Example 3: Write the equation in standard form:  $4x^2 + y^2 - 8x + 4y + 4 = 0$

$$4x^2 - 8x + \square + y^2 + 4y + \square = -4$$

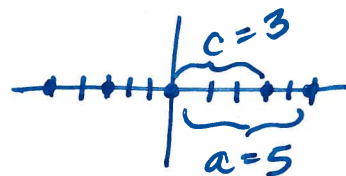
$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = -4 + 4 + 4$$

$$\frac{4(x-1)^2}{4} + \frac{(y+2)^2}{4} = \frac{4}{4}$$

$$\boxed{\frac{(x-1)^2}{1} + \frac{(y+2)^2}{4} = 1}$$

Example 4: Find the equation of the ellipse if the center is  $(0, 0)$ , a focus is  $(3, 0)$ , and a vertex is  $(5, 0)$ .

$$\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$



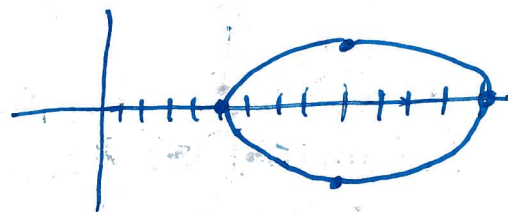
$$b^2 = a^2 - c^2$$

$$b^2 = 25 - 9$$

$$b = 4$$

Example 5: Find the equation of the ellipse if the foci are  $(9, \pm 3)$  and the x-intercepts are ~~±2~~ 5 and 13

$$\frac{(x-9)^2}{16} + \frac{(y-0)^2}{9} = 1$$



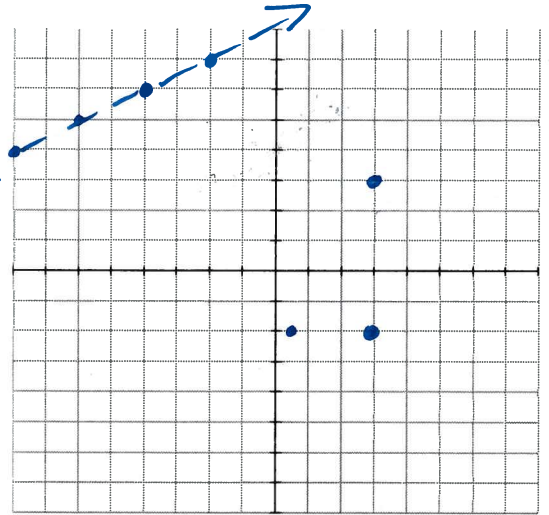


# Ellipse Practice

Point-slope form:  $y - y_1 = m(x - x_1)$

- Find the ordered pair 5 units above  $(3, -2) = (3, 3)$
- Find the ordered pair  $\sqrt{6}$  units to the left of  $(3, -2)$   
 $(3 - \sqrt{6}, -2)$
- Write and graph the equation of the line in point-slope form that has a slope =  $\frac{1}{2}$  and passes through the point  $(-4, 6)$ .

$$y - 6 = \frac{1}{2}(x + 4)$$

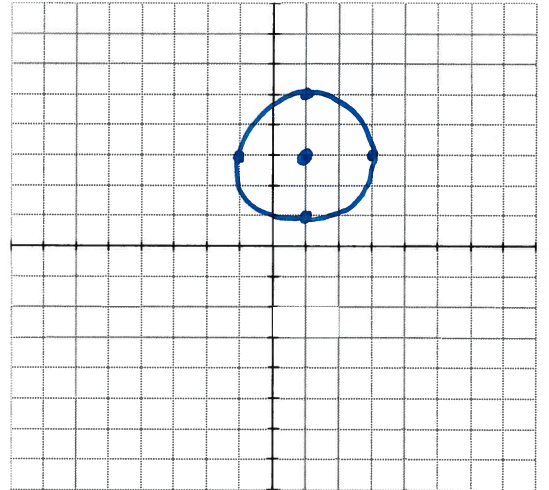


- Find the center, radius, intercepts and sketch the graph of:

$$(x - 1)^2 + (y - 3)^2 = 4$$

Center:  $(1, 3)$

$$r = 2$$



Center, Foci, Vertices

- Discuss the ellipse and sketch its graph:

$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{25} = 1$$

0

Center:  $(3, -2)$

Vertices:  $(3, 3)$   
 $(3, -7)$

Side:  $(1, -2)$   
 $(5, -2)$

Foci:  $(3, -2 + \sqrt{21})$   
 $(3, -2 - \sqrt{21})$

$$a = 5$$

$$b = 2$$

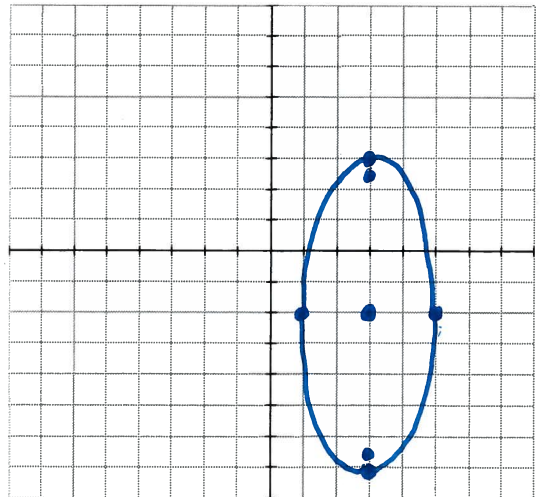
$$c = \sqrt{21}$$

$$b^2 = a^2 - c^2$$

$$4 = 25 - c^2$$

$$+21 = +c^2$$

$$\sqrt{21} = c$$





7. Discuss the ellipse and sketch the graph.

$$b^2 = a^2 - c^2 \quad \frac{2(x+4)^2}{18} + \frac{9(y+2)^2}{18} = 18$$

$$2 = 9 - c^2$$

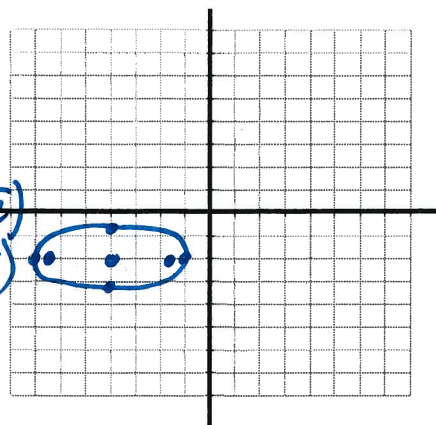
$$c = \sqrt{7} \quad \frac{(x+4)^2}{9} + \frac{(y+2)^2}{2} = 1$$

Center:  $(-4, -2)$

vertices:  $(-7, -2)$   
 $(-1, -2)$

Side vertices:  $(-4, 2+\sqrt{2})$   
 $(-4, 2-\sqrt{2})$

foci:  $(-4+\sqrt{7}, -2)$   
 $(-4-\sqrt{7}, -2)$

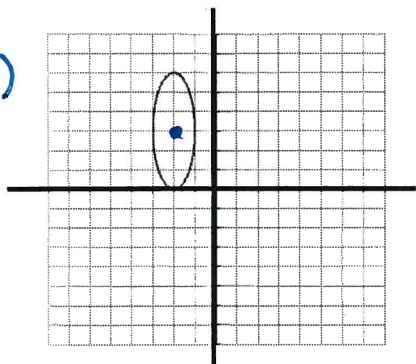


8. Determine the equation of the ellipse shown in the graphs.

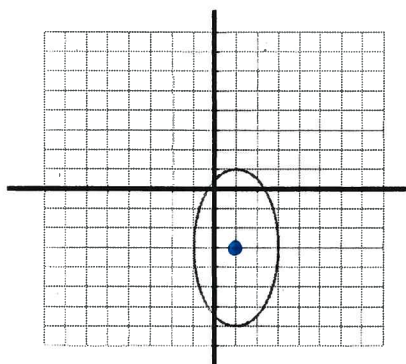
A.)  $\frac{(x+2)^2}{1} + \frac{(y-3)^2}{9} = 1$

B.)  $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{16} = 1$

Center:  $(-2, 3)$   
 $a = 3$   
 $b = 1$



Center:  $(1, -3)$   
 $a = 4$   
 $b = 2$



9. Determine the equation of the ellipse with center at  $(0, 0)$ , focus at  $(-1, 0)$ , and vertex at  $(3, 0)$ .



$$c = 1$$

$$a = 3$$

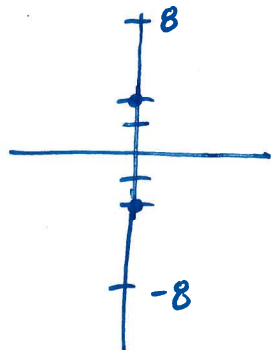
$$b^2 = a^2 - c^2$$

$$b^2 = 9 - 1$$

$$b = \sqrt{8} = 2\sqrt{2}$$

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

10. Determine the equation of the ellipse with foci at  $(0, \pm 2)$  and length of major axis = 8.



$$c = 2$$

$$a = 8$$

$$b^2 = 8^2 - 2^2$$

$$b^2 = 60$$

$$b = 2\sqrt{15}$$

$$\frac{x^2}{60} + \frac{y^2}{64} = 1$$