RATIONALIZING THE DENOMINATOR Name: ______________________

Textbook Chapter 4.5

To rationalize a **Single-Term Denominator** - Multiply both numerator and denominator by the *radical* in the denominator.

1. $\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

2. $\frac{5}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{4(3)} = \frac{5\sqrt{3}}{12}$

To rationalize a denominator with **Two Terms** - Multiply both numerator and denominator by a *conjugate*.

3. $\frac{2}{(3-\sqrt{7})} \cdot \frac{(3+\sqrt{7})}{(3+\sqrt{7})}$

$\frac{6+2\sqrt{7}}{9+\cancel{3\sqrt{7}}-\cancel{3\sqrt{7}}-\sqrt{7}\sqrt{7}}$

$\frac{6+2\sqrt{7}}{9-7} = \frac{6+2\sqrt{7}}{2} = 3+\sqrt{7}$

4. $\frac{3}{(2\sqrt{3}+5)} \cdot \frac{(2\sqrt{3}-5)}{(2\sqrt{3}-5)}$

$\frac{6\sqrt{3}-15}{4(3)-\cancel{10\sqrt{3}}+\cancel{10\sqrt{3}}-25}$

$\frac{-15+6\sqrt{3}}{-13} = \frac{15-6\sqrt{3}}{13}$

To rationalize an **Nth Root Denominator** - Multiply by the base raised to the power of the index minus the exponent.

5. $\frac{1}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} = \sqrt[3]{5^3}$

$\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^3}}$

$\frac{\sqrt[3]{25}}{5}$

6. $\frac{7}{\sqrt[3]{5^2}} \cdot \frac{\sqrt[3]{5^1}}{\sqrt[3]{5^1}} = \sqrt[3]{5^3}$

$\frac{7\sqrt[3]{5}}{\sqrt[3]{5^3}}$

$\frac{7\sqrt[3]{5}}{5}$

Evaluate the expression using a calculator. Round the result to two decimal places.

26. $\sqrt[4]{49} \approx 2.65$ **27.** $\sqrt[9]{19{,}422} \approx 3.00$ **28.** $\sqrt[5]{-122} \approx -2.61$

29. $(215)^{1/5} \approx 2.93$ **30.** $(-15)^{1/3} \approx -2.47$ **31.** $(116)^{1/6} \approx 2.21$

Negative? Positive? Imaginary? What?

Warm-Up. Simplify.

1. $\sqrt{50}$ = $5\sqrt{2}$ $\sqrt{25}\sqrt{2}$ $5\sqrt{2}$	2. $\sqrt{-50}$ = $5i\sqrt{2}$ $i\sqrt{50}$ $i\sqrt{25}\sqrt{2} = i(5)\sqrt{2} = 5i\sqrt{2}$
3. $50^{1/2}$ = $5\sqrt{2}$ $\sqrt{50}$ $\sqrt{25}\sqrt{2}$	4. $50^{-1/2}$ = $\frac{1}{5\sqrt{2}}$ $\frac{1}{50^{1/2}} = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$

NEW NOTES: Nth Roots of Negative Numbers

If there is a negative in an even root, then the answer is imaginary.	If there is a negative in an odd root, then the answer is negative.
5. $\sqrt{-100}$ = $10i$	6. $\sqrt[3]{-8}$ = -2
7. $\sqrt{144}$ = 12	8. $\sqrt[3]{-27}$ = -3
9. $\sqrt{-25}$ = $5i$	10. $\sqrt[3]{64}$ = 4
11. $\sqrt{-16}$ = $4i$	12. $\sqrt[5]{-32}$ = -2
13. $\sqrt{-81}$ = $9i$	14. $\sqrt[3]{-125}$ = -5

Solving Equations using the nth Roots

What values of x do you think will make these equations true?

a) $x^2 = 81$

$x = 9, -9$

$x = \pm 9$

b) $x^3 = 27$

$x = 3$

c) $x^3 = -125$

$x = -5$

* odd roots = only one solution! (no ±)

d) $x^2 = -16$

$x = 4i, -4i$

$(4i)^2 = 16i^2 = -16$ ✓

$(-4i)^2 = 16i^2 = -16$ ✓

Important Facts to remember when solving equations using nth Roots:

- Isolate the exponent first!
- Even Roots: $\pm\sqrt[n]{}$ Odd Roots: *use the sign*
- You only need to use the imaginary # i when you are taking the SQUARE root of a NEGATIVE #
- *CHECK all solutions!!!*

		SIMPLIFY	SOLVE
$\sqrt[2]{x}$	EVEN ROOT	No ± $\sqrt{-1} = i$	±
$\sqrt[3]{x}$	ODD ROOT	No ± $\sqrt[3]{-1} = -1$	No ±

1. $\sqrt{x^2} = \sqrt{49}$ $x = \pm\sqrt{49}$ $x = \pm 7$	2. $\frac{2x^2}{2} = \frac{50}{2}$ $\sqrt{x^2} = \sqrt{25}$ $x = \pm 5$
3. $\sqrt{(x+3)^2} = \sqrt{64}$ $x + 3 = \pm\sqrt{64}$ $x + 3 = \pm 8$ $-3 \quad -3$ $x = -3 \pm 8$ $x = -3+8, \ -3-8$	4. $\frac{3(x-5)^2}{3} = \frac{-27}{3}$ $\sqrt{(x-5)^2} = \sqrt{-9}$ $x - 5 = \pm\sqrt{-9}$ $x - 5 = \pm 3i$ $+5 \quad +5$

$x = 5, -11$

$x = 5 \pm 3i$

$\pm$

5. $2x^4 - 10 = 152$

$+10 \quad +10$

$\frac{2x^4}{2} = \frac{162}{2}$

$\sqrt[4]{x^4} = \sqrt[4]{81}$

$x = \pm\sqrt[4]{81}$

$x = \pm 3$

6. $\sqrt[3]{(x-2)^3} = \sqrt[3]{-125}$

$x - 2 = \sqrt[3]{-125}$

$x - 2 = -5$

$+2 \quad +2$

$x = -3$

7. $\frac{6x^3}{6} = \frac{384}{6}$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

$\pm$

8. $(x-3)^4 + 7 = 632$

$-7 \quad -7$

$\sqrt[4]{(x-3)^4} = \sqrt[4]{625}$

$x - 3 = \pm\sqrt{625}$

$x - 3 = \pm 5$

$+3 \quad +3$

$x = 3 \pm 5$

$x = 3+5, \ 3-5$

$x = 8, -2$

9. $4\left(\frac{1}{4}x^3\right) = (-2)4$

$\sqrt[3]{x^3} = \sqrt[3]{-8}$

$x = \sqrt[3]{-8}$

$x = -2$

$\pm$

10. $2(x-8)^2 - 8 = -108$

$+8 \quad +8$

$\frac{2(x-8)^2}{2} = \frac{-100}{2}$

$\sqrt{(x-8)^2} = \sqrt{-50}$

$x - 8 = \pm\sqrt{-50}$

$x - 8 = \pm\sqrt{-25}\sqrt{2}$

$x - 8 = \pm 5i\sqrt{2}$

$+8 \quad +8$

$x = 8 \pm 5i\sqrt{2}$

11. $\sqrt[5]{(x+1)^5} = \sqrt[5]{100}$

$x + 1 = \sqrt[5]{100}$

$x + 1 = \sqrt[5]{100}$

$-1 \quad -1$

$x = -1 + \sqrt[5]{100}$

$\pm$

12. $\frac{2}{3}\left(\frac{3}{2}x^4\right) = (48) \cdot \frac{2}{3}$

$x^4 = \frac{\cancel{48}^{16}}{1} \cdot \frac{2}{\cancel{3}_1}$

$x^4 = 16 \cdot 2$

$\sqrt[4]{x^4} = \sqrt[4]{32}$

$x = \pm\sqrt[4]{32}$

$x = \pm\sqrt[4]{16}\sqrt[4]{2}$

$x = \pm 2\sqrt[4]{2}$