Ch 1.3 A.7 – Word Problems, Writing an Equation of a Line

DAY 2

Slope-Intercept Form: $y = mx + b$

Standard Form: $ax + by = c$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Write equations in point-slope, slope-intercept, AND standard forms.

1. P = (1, 3); $m = -\frac{2}{5}$ (x_1, y_1)

$y - 3 = -\frac{2}{5}(x - 1)$ → point-slope

$+3 \qquad +3$

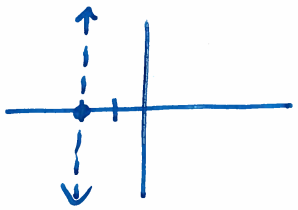
$y = -\frac{2}{5}x + \frac{2}{5} + \frac{3}{1} \cdot \frac{5}{5}$

$(\times 5)$ $y = -\frac{2}{5}x + \frac{17}{5}$ → slope-int

$5y = -2x + 17$

$2x + 5y = 17$ → standard

2. P = (–2, 0); slope undefined (x)



vertical line (VUX)

$x = -2$

3. Containing (–3, 4) and (2, 5)

$m = \frac{5-4}{2-(-3)} = \frac{1}{5}$

$y - 4 = \frac{1}{5}(x + 3)$

$+4 \qquad +4$

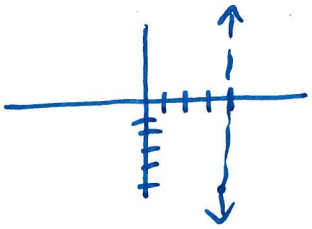
$y = \frac{1}{5}x + \frac{3}{5} + \frac{4}{1} \cdot \frac{5}{5}$

$(\times 5)$ $y = \frac{1}{5}x + \frac{23}{5}$

$5y = x + 23$

$-x + 5y = 23$ → $x - 5y = -23$

4. Vertical; containing (4, –5) (x)



(VUX)

$x = 4$

5. Perpendicular to $x - 2y = -5$ containing (0, 4)

$\perp$: $x - 2y = -5$

$-x \qquad -x$

$\frac{-2y}{-2} = \frac{-x - 5}{-2}$

$y = \frac{x}{2} + \frac{5}{2}$

$y = \frac{1}{2}x + \frac{5}{2}$

$m = \frac{1}{2}$

My line: Point (0, 4)

$m = -2$

$y - 4 = -2(x - 0)$

$+4 \qquad +4$

$y = -2x + 4$

$+2x \quad +2x$

$2x + y = 4$

Translate each word problem into a mathematical equation; then solve.

6. Going into the final exam, which will count as two-thirds of the final grade, Mike has test scores of 86, 80, 84, and 90. What score does Mike need on the final in order to earn a B, which is an average score of 80?

x = Final Exam Score

$$\text{Final Grade} = \frac{2}{3}(\text{Final Exam}) + \frac{1}{3}(\text{Test Avg.})$$

$$80 = \frac{2}{3}x + \frac{1}{3}\left(\frac{86+80+84+90}{4}\right)$$

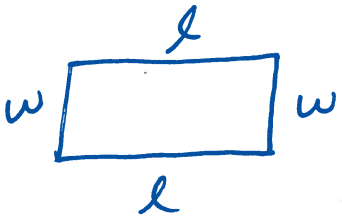
$$3 \cdot \left(80 = \frac{2}{3}x + \frac{1}{3}(85)\right)$$

$$240 = 2x + 85$$
$$-85 \qquad -85$$

$$\frac{155}{2} = \frac{2x}{2}$$

$$\boxed{77.5 = x}$$

7. The perimeter of a rectangle is 60 feet. Find its length and width if the length is 8 feet longer than the width.



w = unknown (w)

$l = w + 8$

$$P = 2l + 2w$$

$$60 = 2(w+8) + 2w$$

$$60 = 2w + 16 + 2w$$

$$60 = 4w + 16$$
$$-16 \qquad -16$$

$$44 = 4w$$

$$11 = w$$

$\boxed{\text{width} = 11 \text{ ft}, \quad \text{length} = 19 \text{ ft}}$

8. Erin, by herself, can paint four rooms in 10 hours. If she hires Alex to help, they can do it together in 6 hours. If she lets Alex work alone, how long will it take him to paint four rooms?

Erin: $\frac{4r}{10} = \frac{10h}{10}$

$\frac{2}{5}r = 1hr$

Together: $\frac{4r}{6} = \frac{6h}{6}$

$\frac{2}{3}r = 1h$

Alex: $1hr = \text{Together} - \text{Erin}$

$1hr = \frac{2}{3}r - \frac{2}{5}r$

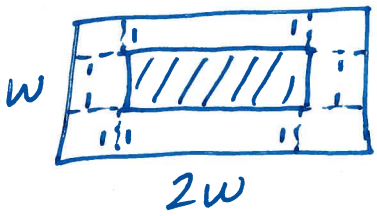
$1hr = \frac{10}{15}r - \frac{6}{15}r$

15. $(1hr = \frac{4}{15}r)$

$15hr = 4r$

4 rooms in 15 hours

9. An open box is to be constructed from a rectangular piece of sheet metal whose length is twice its width by removing a square of side 1 foot from each corner and turning up the edges. If the box is to hold 4 cubic feet, what should be the dimensions of the sheet metal?



w = unknown (w)

$l = 2w$

$V = l \cdot w \cdot h$

$4 = (2w-2)(w-2)(1)$

$4 = 2w^2 - 4w - 2w + 4$

$4 = 2w^2 - 6w + 4$

$-4 \qquad -4$

$0 = 2w^2 - 6w$

$0 = 2w(w-3)$

$w \neq 0$ OR $w = 3$

$w = 3$ feet

$l = 6$ feet

10. An object is propelled vertically upward with an initial velocity of 20 meters per second. The distance s (in meters) of the object from the ground after t seconds is $s = -4.9t^2 + 20t$

(a) When will the object be 15 meters above the ground?

$15 = -4.9t^2 + 20t$

$0 = -4.9t^2 + 20t - 15$

$t = \frac{-20 \pm \sqrt{106}}{-9.8}$

$t \approx 0.99, 3.09$

(b) When will it strike the ground?

$0 = -4.9t^2 + 20t$

$0 = t(-4.9t + 20)$

$t \neq 0$ start OR $-4.9t + 20 = 0$

$t = \frac{20}{-4.9} = 4.08$ sec

(c) Will the object reach a height of 100 meters?

$100 = -4.9t^2 + 20t$

$0 = -4.9t^2 + 20t - 100$

$t = \frac{-20 \pm \sqrt{-1560}}{9.8}$ imag.

No

(d) What is the maximum height? (vertex)

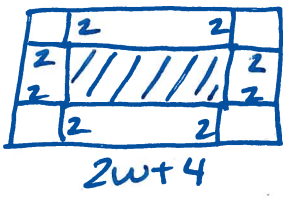
$t = \frac{-b}{2a}$

$t = \frac{-20}{2(-4.9)} = 2.04$

$S = 4.9(2.04)^2 + 20(2.04)$

$S = 61.22\text{ m}$

11. A gardener has 46 feet of fencing to be used to enclose a rectangular garden that has a border 2 feet wide surrounding it. → Perimeter



BASE w, $2w$

$P = 2l + 2w$

$46 = 2(2w+4) + 2(w+4)$

$46 = 4w + 8 + 2w + 8$

$30 = 6w$

$5 = w$

(a) If the length of the garden is to be twice its width, what will be the dimensions of the garden?

$w = 5$ feet

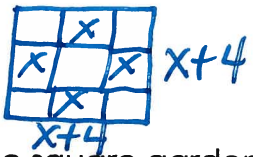
$l = 10$ feet

(b) What is the area of the garden?

$A = l \cdot w$

$A = 50\text{ft}^2$

(c) If the length and width of the garden are to be the same, what would be the dimensions of the garden?



$46 = 4(x+4)$

$46 = 4x + 16$

$30 = 4x$

$x = 7.5\text{ft}$

(d) What would be the area of the square garden?

$A = 5^2$

$A = 7.5^2$

$A = 56.25\text{ft}^2$