

# Factoring Algorithm

Name: \_\_\_\_\_

	Algorithm (instructions)	Example
<b>All Types</b>	<ol style="list-style-type: none"> <li>Put the polynomial in order of decreasing degree (standard form).</li> <li>Factor out the GCF (include any variables!)</li> </ol>	$10 + 7x + x^2$ $x^2 + 7x + 10$ $4x^2 + 14x$ $2x(2x+7)$
<b>Trinomial</b> $x^2 + Bx + C$	<p>If <math>A = 1</math>,</p> <ol style="list-style-type: none"> <li>List the pairs of factors of C.</li> <li>Find the pair that has a sum/difference of the target number.</li> <li>Write the two binomials.</li> </ol>	$x^2 + 7x + 12$ $(x+3)(x+4)$ $12$ $1 \ 12$ $2 \ 6$ $+3+4=7$
<b>Trinomial</b> $x^2 + Bx + C$	<p>If <math>A \neq 1</math>,</p> <ol style="list-style-type: none"> <li>Multiply A and C together and list the pairs of factors.</li> <li>Find the pair that has a sum/difference of the target number.</li> <li>Factor by grouping.</li> </ol> <p>(or factor by trial and error)</p>	$2x^2 - 3x - 20$ $40$ $2x^2 + 5x - 8x - 20$ $1 \ 40$ $2 \ 20$ $x(2x+5) - 4(2x+5)$ $4 \ 10$ $5 \ -8$ $(2x+5)(x-4)$

# BINOMIAL SPECIAL CASE: Difference of Perfect Squares

<p><b>Binomial</b> <math>A^2 - B^2</math></p>	<p>If it is a <u>difference of squares</u>, factor it into conjugates.</p> <p>Formula: <math>A^2 - B^2 = (A+B)(A-B)</math></p>	$x^2 - 100$ $(x+10)(x-10)$
<p><b>Binomial</b> <math>A^2 + B^2</math></p>	<p>If it is a <u>sum of squares</u>, the binomial is <b><u>PRIME</u></b>.</p>	$x^2 + 100$

## Sum of Binomials Cubed

Formula:  $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

## Difference of Binomials Cubed

Formula:  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

<p>Example</p> <p><math>A = X</math> <math>B = 5</math></p> <p><math>(x+5)(x^2 - 5x + 25)</math></p> <p style="margin-left: 100px;">5            0            14P</p>
---

<p>Example</p> <p><math>A = 2x</math> <math>B = 3</math></p> <p><math>(2x+3)(4x^2 - 6x + 9)</math></p>
--

# Factoring

DAY 2

Factor the difference of two squares (DOTS) with the formula:  $A^2 - B^2 = (A + B)(A - B)$

<p>1. <math>1 - 9x^2</math> <math>(1 - 3x)(1 + 3x)</math></p>	<p>2. <math>x^4 - 1</math> <math>(x^2 - 1)(x^2 + 1)</math> <math>(x - 1)(x + 1)(x^2 + 1)</math></p>
---	---

Factor with two variables.

<p>3. <math>6x^2 + 13xy - 5y^2</math></p> $\begin{array}{r} 6x^2 - 2xy + 15xy - 5y^2 \\ \hline 2x \qquad \qquad 5y \end{array} \quad \begin{array}{r} 30 \\ 1 \ 30 \\ -2+15 \\ 3 \ 10 \\ 5 \ 6 \end{array}$ <p><math>2x(3x - y) + 5y(3x - y)</math></p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>(3x - y)(2x + 5y)</math></div>	<p>1. Factor normally without the "y" 2. Put the "y" in at the end</p>
---	--

Factor like a quadratic.

<p>4. <math>4 - 14x^2 - 8x^4</math></p> $\begin{array}{r} -8x^4 - 14x^2 + 4 \\ -2(4x^4 + 7x^2 - 2) \\ -2(4x^4 - x^2 + 8x^2 - 2) \\ \hline -2(x^2(4x^2 - 1) + 2(4x^2 - 1)) \\ -2(4x^2 - 1)(x^2 + 2) \\ -2(2x - 1)(2x + 1)(x^2 + 2) \end{array} \quad \begin{array}{r} 8 \\ -1+8 \\ 2 \ 4 \end{array}$	<p>5. <math>x^6 + 2x^3 + 1</math></p> $\begin{array}{r} (x^3 + 1)(x^3 + 1) \\ (x^3 + 1)^2 \\ [(x + 1)(x^2 - x + 1)]^2 \\ \hline (x + 1)^2 (x^2 - x + 1)^2 \end{array} \quad \begin{array}{r} 1 \\ +1+1 \end{array}$
--	---

Simplify.

<p>6. <math>\frac{(2x - 5) \cdot 3x^2 - x^3 \cdot 2}{(2x - 5)^2}</math></p> $\frac{6x^3 - 15x^2 - 2x^3}{(2x - 5)^2}$ $\frac{4x^3 - 15x^2}{(2x - 5)^2} = \frac{x^2(4x - 15)}{(2x - 5)^2}$	<p>7. <math>3x^2(3x + 4)^2 + x^3 \cdot 2(3x + 4) \cdot 3</math></p> $\frac{3x^2(3x + 4)^2}{3x^2(3x + 4)} + \frac{6x^3(3x + 4)}{3x^2(3x + 4)}$ $3x^2(3x + 4) [(3x + 4) + 2x]$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"><math>3x^2(3x + 4)(5x + 4)</math></div>
--	---

## FACTORIZING OUT THE GCF

Factor each expression completely.

<p>1. <math>5(2x+1)^2 + (5x-6) \cdot 2(2x+1) \cdot 2</math></p> $\frac{5(2x+1)^2}{(2x+1)} + \frac{4(5x-6)(2x+1)}{(2x+1)}$ $(2x+1) [5(2x+1) + 4(5x-6)]$ $(2x+1)(10x+5 + 20x-24) = \boxed{(2x+1)(30x-19)}$	<p>Step 1: Underline each term (separated by + and -)</p>
<p>2. <math>3(4x+5)^2 \cdot 4(5x+1)^2 + (4x+5)^3 \cdot 2(5x+1) \cdot 5</math></p> $\frac{12(4x+5)^2(5x+1)^2}{2(4x+5)^2(5x+1)} + \frac{10(4x+5)^3(5x+1)}{2(4x+5)^2(5x+1)}$ $2(4x+5)^2(5x+1) [6(5x+1) + 5(4x+5)]$ $2(4x+5)^2(5x+1)(30x+6 + 20x+25)$ $\boxed{2(4x+5)^2(5x+1)(50x+31)}$	<p>Step 2: Simplify each term (do not distribute)</p>
	<p>Step 3: Find the GCF</p>
	<p>Step 4: Factor out the GCF (divide and multiply)</p>
	<p>Step 5: Simplify</p>

### PRACTICE

Factor out the GCF.

<p>3. <math>3x^2(8x-3) + x^3 \cdot 8</math></p> $\frac{3x^2(8x-3)}{x^2} + \frac{8x^3}{x^2}$ $x^2 [3(8x-3) + 8x]$ $x^2(24x-9+8x)$ $\boxed{x^2(32x-9)}$	<p>4. <math>4(x+5)^3(x-1)^2 + (x+5)^4 \cdot 2(x-1)</math></p> $\frac{4(x+5)^3(x-1)^2}{2(x+5)^3(x-1)} + \frac{2(x+5)^4(x-1)}{2(x+5)^3(x-1)}$ $2(x+5)^3(x-1) [2 + (x+5)]$ $2(x+5)^3(x-1)(2+x+5)$ $\boxed{2(x+5)^3(x-1)(x+7)}$
---	---