NOTES: COMPLETING THE SQUARE

DAY 15

Textbook Chapter 4.7

OBJECTIVE: Today you will learn about how to factor by completing the square!

SOLVING QUADRATIC EQUATIONS (Completing the Square A=1)

Step 1: Arrange the equation in the form $Ax^2 + Bx = C$

$x^2 + 10x - 3 = 0$

$+3 \quad +3$

$x^2+10x = 3$

Step 2: Determine what value for the third term will make the trinomial a perfect square.

$\left(\frac{10}{2}\right)^2 = 25$

$+25 \quad +25$

Step 3: Add that value to both sides.

$x^2+10x+25 = 28$

Step 4: Simplify (write the trinomial as a binomial squared).

$\sqrt{(x+5)^2} = \sqrt{28}$

Step 5: Take the square root of both sides. (remember the ±)

$x+5 = \pm\sqrt{28}$

$x+5 = \pm\sqrt{4}\sqrt{7}$

$x+5 = \pm 2\sqrt{7}$

$-5 \quad -5$

Step 6: Solve for x (often there are 2 solutions)

$\boxed{x = -5 \pm 2\sqrt{7}}$

---

2. Solve by completing the square.

$x^2 - 4x = 2x + 35$

$-2x \quad -2x$

$x^2-6x = 35$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$+9 \quad +9$

$x^2-6x+9 = 44$

$\sqrt{(x-3)^2} = \sqrt{44}$

$x-3 = \pm\sqrt{4}\sqrt{11}$

$x-3 = \pm 2\sqrt{11}$

$+3 \quad +3$

$\boxed{x = 3 \pm 2\sqrt{11}}$

SOLVING QUADRATIC EQUATIONS (Completing the Square A≠1)

Step 1: Divide both sides by the GCF.

$3x^2 - 6x + 12 = 0$

$-12 \quad -12$

Step 2: Arrange the equation in the form $Ax^2 + Bx = C$

$\frac{3x^2-6x}{3} = \frac{-12}{3}$

$x^2-2x = -4$

Step 3: Determine what value for the third term will make the trinomial a perfect square. Then Add that value to both sides. $\left(\frac{-2}{2}\right)^2 = 1$

$+1 \quad +1$

$x^2-2x+1 = -3$

Step 5: Simplify (write the trinomial as a binomial squared).

$\sqrt{(x-1)^2} = \sqrt{-3}$

Step 6: Take the square root of both sides. (remember the ±)

$x-1 = \pm i\sqrt{3}$

$+1 \quad +1$

Step 7: Solve for x (often there are 2 solutions)

$\boxed{x = 1 \pm i\sqrt{3}}$

CONVERT Standard Form to Vertex Form

Step 1: Factor out the coefficient (2) from the first two terms. $y = 2x^2 + 8x - 5$

$y = 2(x^2+4x) - 5$

Step 2: Complete the square inside the parentheses.

$\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$y = 2(x^2+4x+4-4) - 5$

Step 3: Distribute.

$y = 2((x+2)^2 - 4) - 5$

$y = 2(x+2)^2 - 8 - 5$

Step 4: Simplify.

$\boxed{y = 2(x+2)^2 - 13}$

PRACTICE: SOLVING EQUATIONS

BY COMPLETING THE SQUARE

DAY 15

1. $x^2 + 10x = -23$

$+25 \quad +25$

$(\frac{10}{2})^2 = 25$

$x^2+10x+25 = 2$

$\sqrt{(x+5)^2} = \sqrt{2}$

$x+5 = \pm\sqrt{2}$

$-5 \quad -5$

$\boxed{x = -5 \pm \sqrt{2}}$

2. $x^2 + 2x = -17$

$+1 \quad +1$

$(\frac{2}{2})^2 = 1$

$x^2+2x+1 = -16$

$\sqrt{(x+1)^2} = \sqrt{-16}$

$x+1 = \pm 4i$

$-1 \quad -1$

$\boxed{x = -1 \pm 4i}$

3. $2x^2 - 12x = -14$

$\frac{2}{} \quad \frac{2}{}$

$x^2-6x = -7$

$(\frac{-6}{2})^2 = 9$

$+9 \quad +9$

$x^2-6x+9 = 2$

$\sqrt{(x-3)^2} = \sqrt{2}$

$x-3 = \pm\sqrt{2}$

$+3 \quad +3$

$\boxed{x = 3 \pm \sqrt{2}}$

4. $4x^2 + 16x = -12$

$\frac{4}{} \quad \frac{4}{}$

$x^2+4x = -3$

$(\frac{4}{2})^2 = 4$

$+4 \quad +4$

$x^2+4x+4 = 1$

$\sqrt{(x+2)^2} = \sqrt{1}$

$x+2 = \pm 1$

$-2 \quad -2$

$x = -2 \pm 1$

$x = -2+1, \; -2-1$

$\boxed{x = -1, -3}$

CONVERT TO VERTEX FORM (by completing the square)

5. $y = 2x^2 + 16x - 7$

$(\frac{8}{2})^2 = 16$

$y = 2(x^2+8x) - 7$

$y = 2(x^2+8x+16-16) - 7$

$y = 2[(x+4)^2 - 16] - 7$

$y = 2(x+4)^2 - 32 - 7$

$\boxed{y = 2(x+4)^2 - 39}$

6. $y = 3x^2 - 12x + 8$

$(\frac{-4}{2})^2 = 4$

$y = 3(x^2-4x) + 8$

$y = 3(x^2-4x+4-4) + 8$

$y = 3[(x-2)^2 - 4] + 8$

$y = 3(x-2)^2 - 12 + 8$

$\boxed{y = 3(x-2)^2 - 4}$

SOLVING QUADRATIC EQUATIONS: COMPLETING THE SQUARE

SOLUTIONS = X-INTERCEPTS = ROOTS = ZEROS

STANDARD FORM	$y = x^2 - 2x - 8$	$y = x^2 - 6x - 1$	$y = -2x^2 - 12x - 10$
CONVERT TO VERTEX FORM (complete the square)	$y = x^2 - 2x + 1 - 1 - 8$ $y = (x-1)^2 - 9$ Vertex: (1 , −9)	$y = x^2 - 6x + 9 - 9 - 1$ $y = (x-3)^2 - 10$ Vertex: (3 , −10)	$y = -2(x^2 + 6x) - 10$ $y = -2(x^2 + 6x + 9 - 9) - 10$ $y = -2[(x+3)^2 - 9] - 10$ $y = -2(x+3)^2 + 18 - 10$ $y = -2(x+3)^2 + 8$ Vertex: (−3 , 8)
SOLVE (find the zeros)	$0 = (x-1)^2 - 9$ $+9 \quad +9$ $\sqrt{9} = \sqrt{(x-1)^2}$ $\pm 3 = x - 1$ $+1 \quad +1$ $1 \pm 3 = x$ $1+3,\ 1-3 = x$ $\boxed{4, -2 = x}$	$0 = (x-3)^2 - 10$ $+10 \quad +10$ $\sqrt{10} = \sqrt{(x-3)^2}$ $\pm\sqrt{10} = x - 3$ $+3 \quad +3$ $3 \pm \sqrt{10} = x$ $\boxed{x \approx 6.16, -0.16}$	$0 = -2(x+3)^2 + 8$ $-8 \quad -8$ $\frac{-8}{-2} = \frac{-2(x+3)^2}{-2}$ $\sqrt{4} = \sqrt{(x+3)^2}$ $\pm 2 = x + 3$ $-3 \quad -3$ $-3 \pm 2 = x$ $x = -3+2,\ -3-2$ $\boxed{x = -1, -5}$
Solve by graphing 1 1 2 4 3 9 4 16