Discrete Math
Chapter 6 – review for test!

Name Key

A 1. The number of edges in K_{12} is $\frac{n(n-1)}{2} = \frac{12(12-1)}{2} = 66$

(A) 66
B 12
C 132
D 12!
E none of the above

C 2. The number of Hamilton circuits in K_{12} is $(n-1)!$ $(12-1)!$

A $\frac{12 \times 11}{2}$
B 12
(C) 11!
D 12!
E none of the above

B 3. In a complete graph with 45 edges there is a total of

A 15 vertices
(B) 10 vertices
C 9 vertices
D 11 vertices
E none of the above

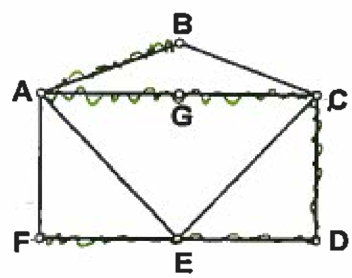
$\frac{n(n-1)}{2} = 45$
$n(n-1) = 90$
$n^2 - n - 90 = 0$
$(n-10)(n+9) = 0$
$n = 10$ ~~$n = -9$~~

OR

Plug in #'s
$\frac{15(15-1)}{2} = 105$ ✗
$\frac{10(10-1)}{2} = 45$ ✓

C 4. The graph to the right

A has a Hamilton path that starts at F and ends at D
B has a Hamilton path that starts at F and ends at E
(C) has a Hamilton path that starts at B and ends at F
D has no Hamilton path
E none of the above



A 5. The brute-force algorithm for solving the Traveling Salesman Problem is

(A) an optimal and inefficient algorithm
B an optimal and efficient algorithm
C an approximate and efficient algorithm
D an approximate and inefficient algorithm
E none of the above

D 6. The nearest neighbor algorithm for solving the Traveling Salesman Problem is

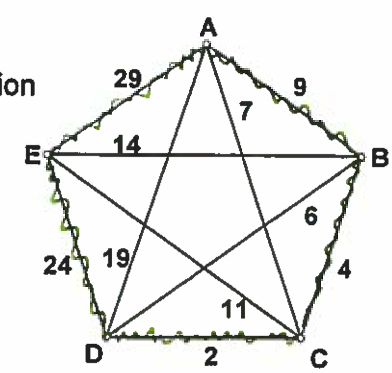
A an optimal and efficient algorithm
B an optimal and inefficient algorithm
C an approximate and inefficient algorithm
(D) an approximate and efficient algorithm
E none of the above

A mail truck must deliver packages to 5 different homes (A, B, C, D, and E). The trip must start and end at A. The graph below shows the distances (in miles) between each home. We want to minimize the total distance traveled.

B 7. The cheapest link algorithm applied to the graph yields the following solution

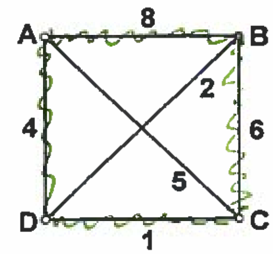
A A, C, D, B, E, A
(B) A, B, C, D, E, A
C A, D, B, E, C, A
D A, E, C, D, B, A
E none of the above

My steps:
① DC 2
② BC 4
not BD (circuit)
not AC (3 from C)
③ AB 9
④ only can: AE 29
⑤ only can: ED 24



A delivery truck must deliver furniture to 4 different locations (A, B, C, and D). The trip must start and end at A. The graph below shows the distances (in miles) between locations. We want to minimize the total distance traveled.

B 8. The nearest neighbor algorithm applied to the graph yields the following solution



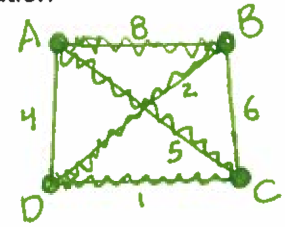
A A, C, B, D, A
(B) A, D, C, B, A
C A, B, D, C, A
D A, D, B, C, A
E none of the above

Start at A
A D C B A
4 1 6 8

A 9. The cheapest link algorithm applied to the graph yields the following solution

(A) A, B, D, C, A
B A, D, B, C, A
C A, C, B, D, A
D A, D, C, B, A
E none of the above

① DC 1
② DB 2
not AD (3 from D)
not BC (circuit
③ AB 8
④ AC (close) 5



A 10. The repetitive nearest neighbor algorithm applied to the graph yields the following solution

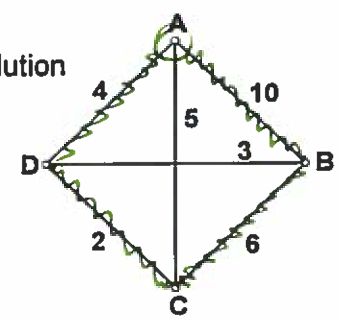
(A) A, B, D, C, A
B A, C, B, D, A
C A, D, C, B, A
D A, D, B, C, A
E none of the above

A	A D B C A	4+2+6+5 = 17
B	B D C A B	2+1+5+8 = 16
C	C D B A C	1+2+8+5 = 16
D	D C A B D	1+5+8+2 = 16

Start at A

A delivery truck must deliver furniture to 4 different locations (A, B, C, and D). The trip must start and end at A. The graph below shows the distances (in miles) between locations. We want to minimize the total distance traveled.

A 11. The nearest neighbor algorithm applied to the graph yields the following solution

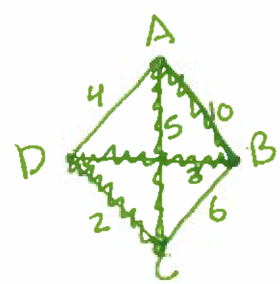


(A) A, D, C, B, A
B A, B, D, C, A
C A, C, B, D, A
D A, D, B, C, A
E none of the above

A D C B A

B 12. The cheapest link algorithm applied to the graph yields the following solution

A A, D, C, B, A
(B) A, B, D, C, A
C A, D, B, C, A
D A, C, B, D, A
E none of the above



13. Given K_8...

a. how many edges are in the graph? $\frac{n(n-1)}{2} = \frac{8(8-1)}{2} =$ **28 edges**

b. how many vertices are in the graph? 8 (Ha! It tells you! K_8 ←)

c. how many distinct Hamilton circuits are in the graph?

$(n-1)! = (8-1)! = 7! =$ **5040 Hamilton Circuits**

A traveling salesman's territory consists of the 5 cities shown on the following mileage chart. The salesman must organize a round trip that starts and ends at Louisville (his hometown) and will pass through each of the other four cities exactly once

Mileage Chart

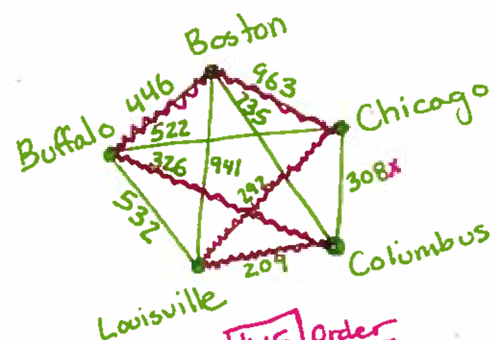
	Boston	Buffalo	Chicago	Columbus	Louisville
Boston	*	446	963	735	941
Buffalo	446	*	522	326	532
Chicago	963	522	*	308	292
Columbus	735	326	308	*	209
Louisville	941	532	292	209	*

__A__ **14.** The nearest neighbor algorithm applied to this problem yields the following solution

A Louisville, Columbus, Chicago, Buffalo, Boston, Louisville
B Louisville, Columbus, Buffalo, Boston, Chicago, Louisville
C Louisville, Boston, Buffalo, Chicago, Columbus, Louisville
D Louisville, Chicago, Buffalo, Boston, Columbus, Louisville
E none of the above

__A__ **15.** The cheapest link algorithm applied to this problem yields the following solution

A Louisville, Columbus, Buffalo, Boston, Chicago, Louisville
B Louisville, Chicago, Buffalo, Boston, Columbus, Louisville
C Louisville, Buffalo, Boston, Columbus, Chicago, Louisville
D Louisville, Boston, Buffalo, Chicago, Columbus, Louisville
E none of the above



#15 Order
209
292
326
446
963

16. How do you find relative error? What exactly are you finding?

weight − optimal / optimal

17. What's the difference between Euler circuits and Hamilton circuits?

Questions 18 – 19 refer to the following situation: A hypothetical management science problem requires us to find the cheapest "supercircuit" in a graph. Three algorithms are available: Algorithm 1, Algorithm 2, and Algorithm 3.

__B__ **18.** Algorithm 1 always (optimal) produces the cheapest supercircuit. The amount of time it takes to carry out Algorithm 1 doubles every time we increase the number of vertices by one. Algorithm 1 is

A an optimal and efficient algorithm
B an optimal and inefficient algorithm
C an approximate and inefficient algorithm
D an approximate and efficient algorithm

__A__ **19.** Algorithm 3 never produces a supercircuit that is off by more than 10% (approximate) from the cheapest supercircuit. The amount of time that it takes to carry out Algorithm 3 is: 1 seconds for a graph with 5 or less vertices, 30 seconds for a graph with 6 vertices, 40 seconds for a graph with 7 vertices, and so on, increasing by 10 seconds every time we add a vertex (from 7 vertices on). Algorithm 3 is

A an approximate and efficient algorithm
B an optimal and inefficient algorithm
C an approximate and inefficient algorithm
D an optimal and efficient algorithm

To find the relative error: $\frac{W - Opt}{Opt}$, where W = weight of the given circuit, and Opt = weight of the optimal circuit!!

_D_ **20.** In trying to solve a certain traveling salesman problem you find a solution with a total length of 250 miles. If the length of the optimal solution is 190, then the relative error of your solution is

A 132%
B 24%
C 124%
D 32%

$\frac{250-190}{190} = 0.3157 = 32\%$

_A_ **21.** In trying to solve a certain traveling salesman problem you find a solution with a total length of 2800 miles. If the length of the optimal solution is 2250 miles, then the relative error of your solution is

A 24%
B 28%
C 20%
D 119%

$\frac{2800-2250}{2250} = 0.2444$

_C_ **22.** In trying to solve a certain Traveling Salesman Problem, you find a solution with a total length of 2500 miles. If the length of the optimal solution is 2000 miles, then the relative error of your solution is

A 125%
B 400%
C 25%
D 20%

$\frac{2500-2000}{2000} = .25$

_B_ **23.** How many different Hamilton circuits would we have to check if we use the brute force algorithm on a graph with 5 vertices? (do not count the same circuit traveled backwards)

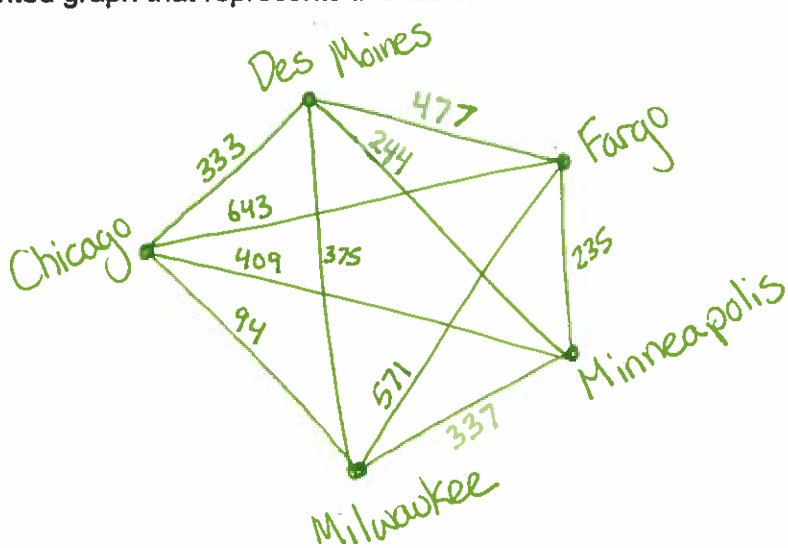
A 6
B 12
C 24
D 4

$K_5 = (5-1)! = 24$ Repeats backwards!

The following question refers to the table of cities and distances listed below.

	Chicago	Des Moines	Fargo	Minne-apolis	Milwau-kee
Chicago	*	333	643	409	94
Des Moines	333	*	477	244	375
Fargo	643	477	*	235	571
Minneapolis	409	244	235	*	337
Milwaukee	94	375	571	337	*

24. Draw a weighted graph that represents this table.



Discrete Math
Chapter 7 – , review

Name Key

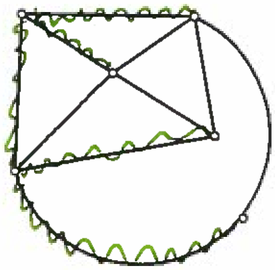
Match each vocabulary word with its corresponding definition.

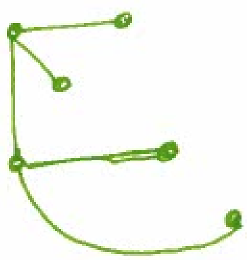
__C__ **1.** Network
__A__ **2.** Subgraph
__D__ **3.** Tree
__E__ **4.** Spanning tree
__B__ **5.** Minimum spanning tree

A a graph whose edges must come from the original graph
B the spanning tree with the least total weight
C a connected graph
D a network with no circuits
E a subgraph that connects all the vertices with no circuits

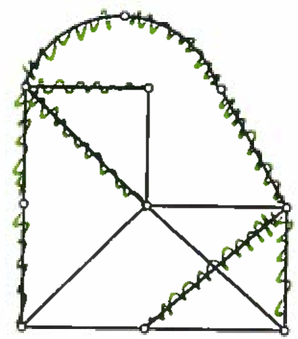
Draw a spanning tree for each graph below.

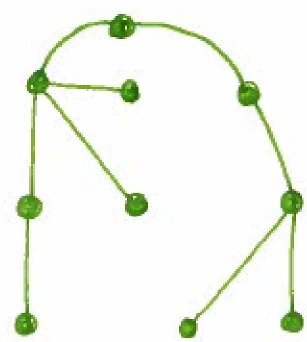
6.





7.



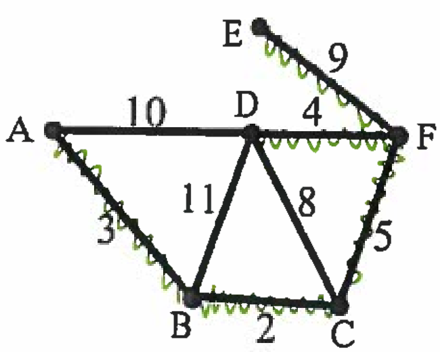


8. A tree contains 17 vertices and __16__ edges.

9. A tree contains __46__ vertices and 45 edges.

Use Kruskal's Algorithm to find the MST. Be sure to identify the weight.

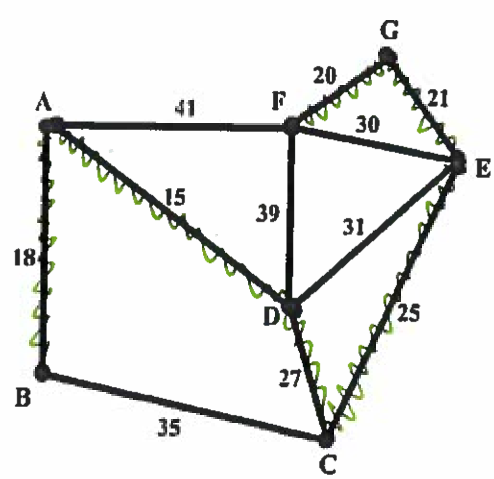
10.



2
3
4
5
9
23

11.

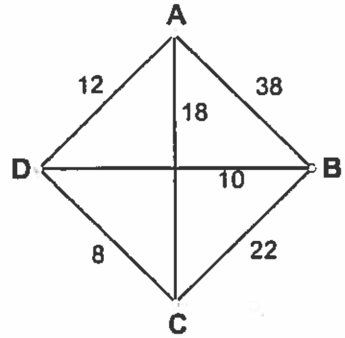
15
18
20
21
25
27
126



Discrete Math
Homework – test review

Name Key

1. For the weighted graph shown in the figure, (i) find the indicated circuit, and (ii) give its cost.



a. The nearest-neighbor circuit for starting vertex A.

A D C B A

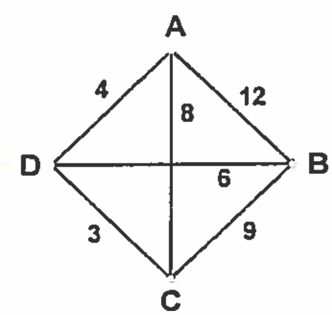
b. The nearest-neighbor circuit for starting vertex B.

B D C A B

c. The nearest-neighbor circuit for starting vertex C.

C D B A C

2. Find the Hamilton circuit obtained by the repetitive nearest-neighbor algorithm, and give the cost for this circuit.



or (A) A D C B A — 4 3 9 12 — 28 ✓

B B D C A B — 6 3 8 12 — 29

or (C) C D A B C — 3 4 12 9 — 28 ✓

D D C A B D — 3 8 12 6 — 29

A D C B A