Exploring Slope of the Tangent Line

Find the slope of the tangent line to the graph of \( f(x) = 2x - 3 \) at \((2, 1)\).

\[
\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 3 - 2x + 3}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2x + 2\Delta x - 2x}{\Delta x}
\]

\[
= \frac{2\Delta x}{\Delta x}
\]

\[
= 2
\]

Using the Slope of the Tangent Line

Find the slopes of the graph of \( g(x) = x^2 \) at the point \((2, 4)\). Find the slope of the tangent line at this point:

\[
\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}
\]

\[
= 2x + 0 = 2x
\]

\[
(\frac{(x+\Delta x)(x+\Delta x)}{x^2 + 2x\Delta x + \Delta x^2})
\]

\[
\text{slope at } x=2 \to 2(2) = 4
\]
\[
\lim_{x \to 0} \frac{1}{x} = \infty
\]

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)
\]

a) Find the slope of the curve at any value \( x \) of \( x \):

\[
\lim_{h \to 0} \frac{1}{h} \cdot \frac{x(X+h)}{X(X+h)}
\]

\[
\lim_{h \to 0} \frac{x-(X+h)}{hx(X+h)}
\]

\[
\lim_{h \to 0} \frac{x-X}{hx(X+h)}
\]

\[
\lim_{h \to 0} \frac{X-X}{x(X+h)} = \frac{-1}{X^2}
\]

b) Where does the slope of the curve equal \(-\frac{1}{4}\)?

\[
\frac{-1}{X^2} = -\frac{1}{4}
\]

\[
4 = X^2
\]

\[
x = \pm 2
\]

Write the equation of the tangent line at \( x = 3 \):

\[
y - y_1 = m(x-x_1)
\]

\[
y - \frac{1}{3} = -\frac{1}{4}(x-3)
\]

\[
m = -\frac{1}{4}
\]

\[
x = \frac{3}{4}
\]

\[
y = \frac{1}{3}
\]

The function \( y = 2t^2 - 1 \) represents the position in feet of an object at time \( t \) seconds. Find the instantaneous rate of change (aka velocity) at \( t = 2 \). Indicate units of measure.

\[
\lim_{\Delta t \to 0} \frac{(2(t+\Delta t)^2 - 1) - (2t^2 - 1)}{\Delta t}
\]

\[
= \lim_{\Delta t \to 0} \frac{2t^2 + 4t\Delta t + 2\Delta t^2 - 1 - 2t^2 + 1}{\Delta t}
\]

\[
= \lim_{\Delta t \to 0} \frac{\Delta t(4t + 2\Delta t)}{\Delta t}
\]

\[
= 4t
\]

At \( t = 2 \) → \( 4t = 8 \) \( \text{ft/sec} \)