DO NOW: Find the horizontal and vertical asymptotes of $f(x)$

$$f(x) = \frac{x^2 - 4}{2x^2 - 5x + 2}$$

$$\lim_{x \to \infty} \frac{x^2 - 4}{2x^2 - 5x + 2}$$

or

$$\lim_{x \to -\infty} \frac{x^2 - 4}{2x^2 - 5x + 2}$$

$$y = \frac{1}{2}$$

$$f(x) = \frac{(x+2)(x-2)}{(2x-1)(x-2)}$$

$$2x-1 = 0 \quad \Rightarrow \quad x = \frac{1}{2}$$
Shortcut for Finding Limits at Infinity

Square Roots with Limits at Infinity

The Squeeze Theorem

Alternate Directions (A Challenging Question from 2008)

| 4. \( \lim_{x \to \infty} \frac{2x^3 + 5}{3x^2 + 1} \) & 5. \( \lim_{x \to \infty} \frac{2x^3 + 5}{3x^2 + 1} \) |
| --- | --- |
| \( = \lim_{x \to \infty} \frac{2 + \frac{5}{x^2}}{3 + \frac{1}{x^2}} \) & \( = \lim_{x \to \infty} \frac{2 + \frac{5}{x^3}}{3 + \frac{1}{x^2}} \) |
| \( = \frac{2}{3} \) & \( \text{top power} \), \( \text{bigger} \), \( \infty \) |
| Same power \( \rightarrow \) ratio \( \rightarrow \frac{2}{3} \) & \( \text{HA rules (million dollar rule)} \) |

Cheater Method: HA rules (million dollar rule)

6. \( \lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 1}} \) \( \rightarrow \) \( x \) & 7. \( \lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 1}} \) \( \rightarrow \) \( \sqrt{x^2} = x \)

\[ \lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 1}} = \frac{3}{\sqrt{2}} \]

\[ = -\frac{3}{\sqrt{2}} \]

Check signs \( \Rightarrow \) top \( - \) bottom +

8. \( \lim \sin x = \text{DNE} \) oscillating & 9. \( \lim \frac{\sin x}{x} = 0 \)

\[ -1 \leq \sin x \leq 1 \]

\[ -\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \]

\[ \lim_{x \to \infty} -\frac{1}{x} \leq \lim_{x \to \infty} \frac{\sin x}{x} \leq \lim_{x \to \infty} \frac{1}{x} \]

\[ 0 \leq \lim_{x \to \infty} \frac{\sin x}{x} \leq 0 \]

What are all horizontal asymptotes of the graph of \( y = \frac{5 + 2x}{1 - 2x} \) in the \( xy \)-plane?

(A) \( y = -1 \) only

(B) \( y = 0 \) only

(C) \( y = 5 \) only

(D) \( y = -1 \) and \( y = 0 \)

(E) \( y = -1 \) and \( y = 5 \)
What is an infinite limit?

Graph \( f(x) = \frac{1}{x^2} \). Fill in the table, then find \( \lim_{x \to 0} \frac{1}{x^2} \).

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
x & -1 & -0.1 & -0.01 & -0.001 & 0 & 0.001 & 0.1 & 1 \\
\hline
f(x) & 1 & 100 & 1,000 & 1,000,000 & \text{und} & 1,000,000 & 100 & 1 \\
\hline
\end{array}
\]

A limit in which \( f(x) \) increases or decreases without bound as \( x \) approaches a number.

EXAMPLE: Graph \( f(x) = \frac{3}{x-2} \). Fill in the table, then find \( \lim_{x \to 2} \frac{3}{x-2} \).

\[
\begin{array}{c}
\lim_{x \to 2} \frac{3}{x-2} = -\infty \\
\lim_{x \to 2} \frac{3}{x-2} = \infty \\
\lim_{x \to 2} \frac{3}{x-2} = \text{DNE}
\end{array}
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.5</th>
<th>1.9</th>
<th>1.99</th>
<th>1.999</th>
<th>2</th>
<th>2.001</th>
<th>2.01</th>
<th>2.1</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-60</td>
<td>-30</td>
<td>-300</td>
<td>-3000</td>
<td>?</td>
<td>und</td>
<td>3000</td>
<td>300</td>
<td>30</td>
</tr>
</tbody>
</table>
Definition of Vertical Asymptote

The line \( x = a \) is a vertical asymptote of \( f(x) \) if...

Vertical asymptotes (a lot of the time) happen when...

\[
\text{denominator} = 0 \quad \text{when} \quad \text{num} \neq 0
\]

Determine the vertical asymptote(s) of

\[
g(x) = \frac{x^2 + 2x - 8}{x^2 - 4} = \frac{(x+4)(x-2)}{(x+2)(x-2)}
\]

Find the following limits.

\[
\lim_{x \to 1} \frac{x^2 - 3x}{x - 1} = \frac{0}{0} \quad \text{undetermined}
\]

\[
\lim_{x \to 1} \frac{1^2 - 3(1)}{1 - 1} = \frac{0}{0} \quad \text{undetermined}
\]

Let \( \lim_{x \to a} f(x) = \infty \) and \( \lim_{x \to a} g(x) = L \).

\[
\lim_{x \to a} [f(x) \pm g(x)] = \infty \pm L
\]

\[
\lim_{x \to a} [f(x) \cdot g(x)] = \infty (L) \quad L > 0
\]

\[
\lim_{x \to a} [f(x) / g(x)] = \infty (L) \quad L < 0
\]

\[
\lim_{x \to a} \frac{g(x)}{f(x)} = \frac{L}{\infty}
\]

If the graph of \( y = \frac{ax + b}{x + c} \) has a horizontal asymptote \( y = 2 \) and a vertical asymptote \( x = -3 \), then \( a + c = \)

(A) -5  (B) -1  (C) 0  (D) 1  (E) 5

\[
a = 2 \quad \text{and} \quad x + \sqrt{3} = 0 \quad \text{when} \quad x = -3
\]
If your oven is at 250 degrees and you turn it off, is there ever an instant when the oven temperature is 170 degrees?

Yes

170 is between 250 & room temp (74°)

temp. is continuous
If you remove marbles from a bag one at a time, must there always come a time when the bag contains exactly half the number of marbles it began with?

3 marbles

\[ \frac{1}{2} \text{ marbles} \quad \text{is between all and none} \]

3 ---
2 ---
1 ---
0 ---

not continuous
Next class: Review & Quiz on 2.1-2.3

* I will post the review if you want to start on it before next class

- limits w/ tables (calculator)
- limits w/ graphs
- limits w/ algebra
  - factor
  - expand
  - special trig
- limits w/ infinity
- HA rules
  +∞ w/ check signs