6. Simplify the following expressions. Please leave your answer in factored form.

\[
\frac{x^2 - x}{x^2 + 7x - 8} = \frac{x(x-1)}{(x+8)(x-1)} = \frac{x}{x+8}
\]

\[
\frac{14x^2 + 40x - 6}{x^3 - 2x^2 - 15x} = \frac{2(7x^2 + 20x - 3)}{x(x^2 - 2x - 15)} = \frac{2(7x-1)(x+3)}{x(x-5)(x+3)} = \frac{2(7x-1)}{x(x-5)}
\]

\[
\frac{x-2}{\sqrt{x}-\sqrt{2}} = \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \frac{x-2}{\sqrt{x}+\sqrt{2}}
\]

\[
\frac{x-7}{\sqrt{9-x}-\sqrt{2x-12}} = \frac{\sqrt{9-x}+\sqrt{2x-12}}{\sqrt{9-x}+\sqrt{2x-12}} = \frac{\sqrt{9-x}+\sqrt{2x-12}}{\sqrt{9-x}+\sqrt{2x-12}} = \frac{x-7}{\sqrt{9-x}+\sqrt{2x-12}}
\]

\[
\frac{1}{4} + \frac{1}{x+2} - \frac{4(x+2)}{4(x+2)} = \frac{(x+2) - 4}{(x-2)4(x+2)} = \frac{x-2}{4(x-2)(x+2)} = \frac{1}{4(x+2)}
\]

\[
\frac{x-2}{x(x+2)(x-2)} = \frac{2x}{x(x+2)(x-2)} = \frac{2}{x(x+2)(x-2)}
\]
Test Talk (75ish points)

* parent functions matching (graph & equation)
* characteristics (zeros, VA, HA, hole, domain)
* solve (factor when a is not 1, one with y')
* evaluating functions (composition w/ table, graph, equation/piecewise function)
* simplify (factor, common denominator, conjugate)
* BONUS

NOTE: Graded Problem Set due next class (at the BEGINNING of class)
Notes: 2.1 Limits (with Tables & Graphs)

Consider the function \( f(x) = \frac{x^3 - 1}{x - 1} \).

- \( \lim_{x \to 2} f(x) = 7 \)
- \( \lim_{x \to 2} \frac{x^3 - 1}{x - 1} = 7 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.9</th>
<th>1.99</th>
<th>1.999</th>
<th>2</th>
<th>2.001</th>
<th>2.01</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>6.51</td>
<td>6.9501</td>
<td>6.995001</td>
<td>7</td>
<td>7.005001</td>
<td>7.0501</td>
<td>7.51</td>
</tr>
</tbody>
</table>

What value does the function approach as \( x \) approaches \( x = 2 \)? __7__

- Is \( f(2) \) defined? **yes**
- Does the \( f(2) \) match the value the function approaches as \( x \) approaches? **yes**

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.9</th>
<th>0.99</th>
<th>0.999</th>
<th>1</th>
<th>1.001</th>
<th>1.01</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2.71</td>
<td>2.9701</td>
<td>2.997001</td>
<td>undf.</td>
<td>3.003001</td>
<td>3.0301</td>
<td>3.31</td>
</tr>
</tbody>
</table>

What value does the function approach as \( x \) approaches \( x = 1 \)? __3__

- Is \( f(1) \) defined? **no**
- Does the \( f(1) \) match the value the function approaches as \( x \) approaches? **no**

Sketch a graph of the function:
Definition of a limit

Describe how the outputs of a function behave as the inputs approach some particular value.

\[ \lim_{x \to c} f(x) = L \]

How to say the notation above: “the limit as \( x \) approaches \( c \) of \( f \) of \( x \) equals \( L \).”

Example: Limits

Find \( \lim_{x \to 0} \frac{\sin x}{x} \). Justify your answer using a table and/or graph.

\[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-0.2</th>
<th>-0.1</th>
<th>-0.01</th>
<th>0</th>
<th>0.01</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0.993</td>
<td>0.998</td>
<td>0.999</td>
<td>undef</td>
<td>0.999</td>
<td>0.998</td>
<td>0.993</td>
</tr>
</tbody>
</table>

Important note!

The value of \( f(c) \) DOES NOT matter when finding \( \lim_{x \to c} f(x) \).

\[
\begin{align*}
  f(x) &= \frac{x^2 - 1}{x - 1} \\
  g(x) &= \begin{cases} 
    \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\
    1 & \text{if } x = 1 
  \end{cases} \\
  h(x) &= x + 1 \\
\end{align*}
\]

\[ \begin{align*}
  \lim_{x \to 1} f(x) &= 2 \\
  \lim_{x \to 1} g(x) &= 2 \\
  \lim_{x \to 1} h(x) &= 2 
\end{align*} \]
One-Sided Limits

\[ \lim_{x \to c^-} f(x) \text{ means limit from the right} \]

\[ \lim_{x \to c^+} f(x) \text{ means limit from the left} \]

Given the graph of \( y = f(x) \), find the limit.

<table>
<thead>
<tr>
<th>( \lim_{x \to -5} f(x) )</th>
<th>( \lim_{x \to -2} f(x) )</th>
<th>( \lim_{x \to 1} f(x) )</th>
<th>( f(-2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \lim_{x \to -2} f(x) )</th>
<th>( \lim_{x \to -1} f(x) )</th>
<th>( \lim_{x \to 1} f(x) )</th>
<th>( f(-1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \lim_{x \to 1} f(x) )</th>
<th>( \lim_{x \to 3} f(x) )</th>
<th>( \lim_{x \to 1} f(x) )</th>
<th>( f(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-3</td>
<td>does not exist</td>
<td>DNE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \lim_{x \to 3} f(x) )</th>
<th>( \lim_{x \to -2} f(x) )</th>
<th>( \lim_{x \to 4} f(x) )</th>
<th>( f(4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>undefined</td>
</tr>
</tbody>
</table>

Theorem: One-Sided and Two-Sided Limits

A function \( f(x) \) has a limit as \( x \) approaches \( c \) if and only if...

the limit from the left and right are equal

Symbolically:

\[ \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \]
1. Behavior

Find \( \lim_{x \to 0} f(x) \) where 
\[
f(x) = \begin{cases} 
-x - 3, & x \leq 0 \\
x^2, & x > 0 
\end{cases}
\]

2. Behavior

Find \( \lim_{x \to 0} \frac{1}{x^2} \)

3. Behavior

Find \( \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{2}{\pi} )</th>
<th>( \frac{2}{3\pi} )</th>
<th>( \frac{2}{5\pi} )</th>
<th>( \frac{2}{7\pi} )</th>
<th>( \frac{2}{9\pi} )</th>
<th>\ldots</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>\ldots</td>
<td>?</td>
</tr>
</tbody>
</table>

Use the graph of the function \( f \) to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

\[
\begin{array}{ccc}
\lim_{x \to -2} f(x) & \lim_{x \to 2} f(x) & \lim_{x \to -\infty} f(x) \\
\lim_{x \to -2} f(x) & \lim_{x \to 2} f(x) & \lim_{x \to -\infty} f(x) \\
\lim_{x \to 0} f(x) & \lim_{x \to 4} f(x) & \lim_{x \to 2} f(x)
\end{array}
\]