## Unit 7 | Integrals

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### Vocab
- Approximate, overestimate, underestimate
- Evaluate
- Average
- Bounded by

### Notes Checks
- [ ] Section 7.1
- [ ] Section 7.2
- [ ] Section 7.3
- [ ] Section 7.4
- [ ] Section 7.5
- [ ] Section 7.6
- [ ] Section 7.7

### Problem Sets
- [ ] Round 2 Problem Set Due
- [ ] Graded Problem Set Due

### Qualified for Round 2?
- o Yes
- o No
Notes: 7.3 Definite Integrals

<table>
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<th>Definite integrals</th>
<th>The area under a curve is equal to the definite integral bounded by curve &amp; x-axis.</th>
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<tr>
<td>Notation:</td>
<td>$\int_{a}^{b} f(x) , dx$</td>
</tr>
<tr>
<td></td>
<td>integrand</td>
</tr>
<tr>
<td></td>
<td>bounds (limits)</td>
</tr>
<tr>
<td>Definition of a definite integral</td>
<td>Let $f$ be a function defined on $[a, b]$. If $\lim_{\Delta x \to 0} \sum f(c_i) \Delta x_i$ exists, then $f$ is integrable on $[a, b]$ and the limit is given by $\int_{a}^{b} f(x) , dx$ (the definite integral of $f$ over $[a, b]$).</td>
</tr>
<tr>
<td>Continuity implies integrability</td>
<td>All continuous functions are integrable. This means if $f$ is continuous on $[a, b]$, then its definite integral over $[a, b]$ exists.</td>
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<tr>
<td>What is signed area?</td>
<td>Evaluate the definite integral $\int_{-2}^{1} 2x , dx$ using a right-endpoint Riemann sum with 6 subintervals.</td>
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<tr>
<td></td>
<td>$\Delta x = \frac{b-a}{n} = \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$f(x) = 2x$</td>
</tr>
<tr>
<td></td>
<td>interval $[-2, 1]$</td>
</tr>
<tr>
<td></td>
<td>$x$</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>-4</td>
</tr>
<tr>
<td>$\int_{-2}^{1} 2x , dx \approx \frac{1}{2}(-3-2-1+0+1+2) = -\frac{3}{2}$</td>
<td></td>
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Signed area (area under the x-axis is negative)
EXAMPLE: Evaluate $\int_{-2}^{1} 2x \, dx$ using a right-endpoint Riemann sum with $n$ rectangles.

$\Delta x = \frac{b-a}{n} = \frac{1 - (-2)}{n} = \frac{3}{n}$

$x_i = a + i \Delta x = -2 + i \left( \frac{3}{n} \right) = -2 + \frac{3i}{n}$

$f(x_i) = 2 \left( -2 + \frac{3i}{n} \right)$

$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{3}{n} \cdot 2 \left( -2 + \frac{3i}{n} \right)$

Set up the Riemann Sum equal to $\int_{-2}^{1} x \, dx$:

(A) $\lim_{n \to \infty} \sum_{k=1}^{n} \left( 2 + \frac{k}{n} \right)^2 \frac{1}{n}$

(B) $\lim_{n \to \infty} \sum_{k=1}^{n} \left[ 2 + \frac{k}{n} \right]^2 \frac{3}{n}$

(C) $\lim_{n \to \infty} \sum_{k=1}^{n} \left( 2 + \frac{2k}{n} \right)^2 \frac{1}{n}$

(D) $\lim_{n \to \infty} \sum_{k=1}^{n} \left[ 2 + \frac{3k}{n} \right]^2 \frac{3}{n}$
Using area (and geometry) to our advantage

Sketch the region corresponding to each definite integral. Then use your math skills to evaluate each integral.

\[
\int_{1}^{4} f(x) \, dx = 2 \cdot (4) = 8
\]

\[
\int_{0}^{x+2} f(x) \, dx = 3 \cdot (2) + \frac{1}{2} \cdot (3) \cdot (3)
\]

\[
= 6 + \frac{9}{2}
\]

\[
= \frac{21}{2} = 10.5
\]

\[
\int_{-2}^{\sqrt{4-x^2}} f(x) \, dx
\]

\[
y^2 = 4 - x^2
\]

\[
x^2 + y^2 = 4
\]

\[
\frac{\pi}{2} \cdot (2)^2 = 2\pi
\]

Using Properties of Integrals

- \[\int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx\]
- \[\int_{a}^{b} f(x) \, dx = \int_{a}^{c} f(x) \, dx + \int_{c}^{b} f(x) \, dx\]
- \[\int_{a}^{b} k f(x) \, dx = k \int_{a}^{b} f(x) \, dx\]

Given \[\int_{0}^{a} f(x) \, dx = 4\] and \[\int_{a}^{b} f(x) \, dx = -1\], find

1. \[\int_{0}^{b} f(x) \, dx = \int_{0}^{3} f(x) \, dx + \int_{3}^{b} f(x) \, dx = 4 + (-1) = 3\]
2. \[\int_{a}^{b} f(x) \, dx = 0\]
3. \[\int_{a}^{b} f(x) \, dx = -\int_{3}^{b} f(x) \, dx = -(-1) = 1\]
4. \[\int_{a}^{b} -5 f(x) \, dx = -5 \int_{3}^{b} f(x) \, dx = -5(-1) = 5\]
Let $f$ be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given to the right. Let $g$ be the function given by $g(x) = \int_x^3 f(t)\,dt$.

Find the values of $g(2)$ and $g(-2)$.

$$g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2} \left(1\right) \left(\frac{1}{2}\right) = -\frac{1}{4}$$

$$g(-2) = \int_{-2}^{-1} f(t)\,dt = -\int_{-2}^1 f(t)\,dt$$

$$= -\left(\frac{1}{2} \left(1\right) \left(3\right) - \frac{\pi}{2} \left(1\right)^2\right)$$

$$= \frac{\pi}{2} - \frac{3}{2}$$

The figure above shows the graph of the piecewise-linear function $f$. For $-4 \leq x \leq 12$, the function $g$ is defined by $g(x) = \int_x^3 f(t)\,dt$.

Find $g(2)$ and $g(10)$. 

\[O = \pi r^2\]

\[\square = \frac{\pi}{2} r^2\]