Notes: 6.3 L’Hôpital’s Rule

AP Style Problems

\[ \lim_{h \to 0} \frac{\tan 3(x+h) - \tan 3x}{h} = \frac{f(x) = \tan 3x}{f'(x) = \sec^2 (3x) \cdot 3} \]

(A) 0 \quad (B) \sec^2(3x) \quad (C) \tan(3x) \quad (D) 3\cot(3x) \quad (E) nonexistent

\[ \lim_{h \to 0} \tan 3(x+h) - \tan 3x = 0 \]
\[ \lim_{h \to 0} h = 0 \]

by L’Hôpital’s Rule

\[ \lim_{h \to 0} 3\sec^2(3(x+h)) - 0 = 3\sec^2(3x) \]

What is \( \lim_{x \to 0} \frac{e^{2x} - 1}{\tan x} \)?

(A) \(-1\) \quad (B) 0 \quad (C) 1 \quad (D) \frac{2}{1} \quad (E) The limit does not exist.

\[ \lim_{x \to 0} e^{2x} - 1 = e^0 - 1 = 0 \]
\[ \lim_{x \to 0} \tan x = 0 \]

by L’Hôpital’s Rule

\[ \lim_{x \to 0} \frac{e^{2x} - 1}{\tan x} = \lim_{x \to 0} \frac{2e^{2x}}{\sec^2 x} = \frac{2e^0}{\sec^2 0} = \frac{2}{1} = 2 \]

\[ \lim_{\theta \to 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} \]

(A) 0 \quad (B) \frac{1}{8} \quad (C) \frac{1}{4} \quad (D) 1 \quad (E) nonexistent

\[ \lim_{\theta \to 0} 1 - \cos \theta = 1 - \cos 0 = 1 - 1 = 0 \]
\[ \lim_{\theta \to 0} 2\sin^2 \theta = 2\sin^2 0 = 0 \]

by L’Hôpital’s Rule

\[ \lim_{\theta \to 0} \frac{1 - \cos \theta}{2\sin^2 \theta} = \lim_{\theta \to 0} \frac{\sin \theta}{4\sin \theta \cos \theta} = \frac{1}{4} \]
# Notes: 6.4 Linearization

**Linearization AKA Tangent Line Approximation**

Find the linearization of the function $f(x) = 1 + \sin x$ at the point $(0, 1)$. Then compare the $y$-values of the linear function with those of $f(x)$.

- $f'(x) = \cos x$
- $f'(0) = \cos 0 = 1$

- $y - y_1 = m(x - x_1)$
- $y - 1 = 1(x - 0)$
- $y = x + 1$
- $f(x) \approx x + 1$
- around $x = 0$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-0.5$</th>
<th>$-0.1$</th>
<th>$-0.01$</th>
<th>$0$</th>
<th>$0.01$</th>
<th>$0.1$</th>
<th>$0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 1 + \sin x$</td>
<td>0.520</td>
<td>0.900</td>
<td>0.990</td>
<td>1.01</td>
<td>1.099</td>
<td>1.479</td>
<td></td>
</tr>
<tr>
<td>$y = x + 1$</td>
<td>0.5</td>
<td>0.9</td>
<td>0.99</td>
<td>1.01</td>
<td>1.1</td>
<td>1.5</td>
<td></td>
</tr>
</tbody>
</table>

**AP Style Question**

The function $f$ is twice-differentiable with $f(3) = 2$, $f'(3) = 5$, and $f''(3) = -1$.

What is the value of the approximation of $f(4)$ using the line tangent to the graph of $f$ at $x = 3$?

- $y - 2 = 5(x - 3)$
- $f(x) \approx 2 + 5(x - 3)$
- $f(4) \approx 2 + 5(4 - 3) = 7$

If the line tangent to the graph from above is used to find an approximation to a zero of $f$, that approximation is

- a) 0.4
- b) 0.5
- c) 2.6
- d) 3.4
- e) 5.5

- $y - 2 = 5(x - 3)$
- $0 - 2 = 5(x - 3)$
- $-2 = 5(x - 3)$
- $x = \frac{15}{5} - \frac{2}{5} = \frac{13}{5} = \frac{2}{10}$
- $= 2.6$
Using Tangent Lines to Approximate

The approximate value of \( y = \sqrt{4 + \sin x} \) at \( x = 0.12 \), obtained from the line tangent to the graph at \( x = 0 \) is

a) 2.00  
b) 2.03  
c) 2.06  
d) 2.12  
e) 2.24

Choosing your own \( x \)-value

Find the linearization of the function \( f(x) = \sqrt{5 - x} \) at an appropriate \( x \)-value to approximate \( \sqrt{3.98} \) and \( \sqrt{4.05} \).

\[
\begin{align*}
  f'(x) &= \frac{-1}{2\sqrt{5-x}} \quad \text{at } x=1 \\
  f'(1) &= \frac{-1}{2\sqrt{4}} = -\frac{1}{4} \\
  y - 2 &= -\frac{1}{4}(x - 1) \\
  f(x) &= \sqrt{5-x} \approx 2 - \frac{1}{4}(x - 1) \\
  \sqrt{3.98} &= f(1.02) \approx 2 - \frac{1}{4}(1.02 - 1) = 1.995 \\
  \sqrt{4.05} &= f(0.95) \approx 2 - \frac{1}{4}(0.95 - 1) = 2.0125
\end{align*}
\]

Making good approximations

Should we make a new tangent line if we wanted to approximate \( \sqrt{1.2} \)? Why or why not?

appropriate \( x \)-value: \( x = 4 \)