Notes: 6.3 L’Hospital’s Rule

AP Style Problems

The limit \( \lim_{h \to 0} \frac{\tan 3(x+h) - \tan 3x}{h} \) is defined as \( f'(x) = 3\sec^2 3x \).

- \( f(x) = \tan 3x \)
- \( \Delta x \) defn
- \( f'(x) = 3\sec^2 3x \)

\[ \lim_{h \to 0} \frac{\tan 3(x+h) - \tan 3x}{h} = 0 \]
\[ \lim_{h \to 0} h = 0 \]

by L’Hôpital’s Rule

\[ \lim_{h \to 0} \frac{3\sec^2(3(x+h)) - 0}{h} = 3\sec^2(3x) \]

What is \( \lim_{x \to \infty} e^{2x} - 1 \)?

- \( A \) -1
- \( B \) 0
- \( C \) 1
- \( D \) 2
- \( E \) The limit does not exist.

\[ \lim_{x \to \infty} e^{2x} - 1 = e^0 - 1 = 0 \]

\[ \lim_{x \to 0} \tan x = 0 \]

by L’Hôpital’s Rule

\[ \lim_{x \to 0} \frac{e^{2x} - 1}{\tan x} = \lim_{x \to 0} \frac{2e^{2x}}{\sec^2 x} = \frac{2e^0}{\sec^2 0} = \frac{2}{1} = 2 \]

\[ \lim_{\theta \to 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} \]

- \( A \) 0
- \( B \) \( \frac{1}{8} \)
- \( C \) \( \frac{1}{4} \)
- \( D \) 1
- \( E \) non-existent

\[ \lim_{\theta \to 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} = \lim_{\theta \to 0} \frac{\sin \theta}{\theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{1}{4} \]
Notes: 6.4 Linearization

Linearization AKA Tangent Line Approximation

Find the linearization of the function \( f(x) = 1 + \sin x \) at the point \((0,1)\). Then compare the y-values of the linear function with those of \( f(x) \).

\[ f'(x) = \cos x \]
\[ f'(0) = \cos(0) = 1 \]

\[ y - y_1 = m(x - x_1) \]
\[ y - 1 = 1(x - 0) \]
\[ y = 1 + x \]

\[ f(x) \approx 1 + x \]
around \( x = 0 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-0.5)</th>
<th>(-0.1)</th>
<th>(-0.01)</th>
<th>(0)</th>
<th>(0.01)</th>
<th>(0.1)</th>
<th>(0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 1 + x )</td>
<td>0.520</td>
<td>0.900</td>
<td>0.990</td>
<td>1.009</td>
<td>0.999</td>
<td>1.419</td>
<td></td>
</tr>
</tbody>
</table>

Comparing y-values

The function \( f \) is twice-differentiable with \( f'(3) = 2, f''(3) = 5, \) and \( f'''(3) = -1 \).

What is the value of the approximation of \( f(4) \) using the line tangent to the graph of \( f \) at \( x = 3 \)?

\[ y - y_1 = m(x - x_1) \]
\[ y - 2 = 5(x - 3) \]
\[ y = 2 + 5(x - 3) \]
\[ f(x) \approx 2 + 5(x - 3) \]

\[ f(4) \approx 2 + 5(4 - 3) = 2 + 5 = 7 \]

If the line tangent to the graph from above is used to find an approximation to a zero of \( f \), that approximation is

- (b) 0.4
- (c) 0.5
- (d) 2.5
- (e) 3.4
- (f) 5.5

\[ y - 2 = 5(x - 3) \]
\[ 0 - 2 = 5(x - 3) \]
\[ 0 - 2 = 5x - 15 \]
\[ 5x = 15 - 2 \]
\[ 5x = 13 \]
\[ x = \frac{13}{5} \]

\[ x = \frac{13}{5} - \frac{2}{5} = \frac{11}{5} \cdot \frac{2}{2} = \frac{22}{10} = 2.4 \]
<table>
<thead>
<tr>
<th>Using Tangent Lines to Approximate</th>
<th>The approximate value of $y = \sqrt{\frac{1}{4} + \sin x}$ at $x = 0.12$, obtained from the line tangent to the graph at $x = 0$ is</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 2.00</td>
<td>b) 2.03</td>
</tr>
<tr>
<td>c) 2.06</td>
<td>d) 2.12</td>
</tr>
<tr>
<td>e) 2.24</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Choosing your own $x$-value</th>
<th>Find the linearization of the function $f(x) = \sqrt{5 - x}$ at an appropriate $x$-value to approximate $\sqrt{3.98}$ and $\sqrt{4.05}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x) = \frac{-1}{2\sqrt{5-x}}$</td>
<td>$f'(1) = \frac{-1}{2\sqrt{4}} = \frac{-1}{4}$ \hspace{1cm} $f(1) = \sqrt{5-1} = \sqrt{4} = 2$ \hspace{1cm} $x = 1$ \hspace{1cm} $f(1) = 2$</td>
</tr>
</tbody>
</table>

\[
y - 2 = -\frac{1}{4}(x-1) \\
f(x) - \sqrt{5 - x} \approx 2 - \frac{1}{4}(x-1) \quad \text{around } x = 1
\]

\[
\sqrt{3.98} = f(1.02) \approx 2 - \frac{1}{4}(1.02 - 1) = 1.995 \\
\sqrt{4.05} = f(0.95) \approx 2 - \frac{1}{4}(0.95 - 1) = 2.0125
\]

Making good approximations

Should we make a new tangent line if we wanted to approximate $\sqrt{1.2}$? Why or why not?

appropriate $x$-value: 4