DO NOW: Find a partner!

critical #s

\[ f(x) \rightarrow \pm 1 \]
\[ g(x) \rightarrow 0, -2 \]
\[ f'(x) \]

- \[ f \text{ has a relative min at } x = 1 \text{ because } f' \text{ changes from negative to positive} \]
- \[ f \text{ has a relative max at } x = -1 \text{ because } f' \text{ changes from positive to negative} \]
- \[ f \text{ is increasing } (-\infty, -1) \cup (1, \infty) \text{ because } f' > 0 \]
- \[ f \text{ is decreasing } (-1, 1) \text{ because } f' < 0 \]
$g'(x)$

-2 6

g inc: $(-2, 6)$  
g dec: $(-\infty, -2)$ $[6, \infty)$  
g max: 6  
g min: -2
Notes: 5.5 Connecting $f'$ and $f''$ with the Graph of $f$

The Second Derivative Test

Interpreting the Graph of $f''$

AP Free Response:
2015 #5

The figure above shows the graph of $f''$, the derivative of a twice-differentiable function $f$, on the interval $[-3, 4]$. The graph of $f''$ has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of $f''$ on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which $f$ has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of $f$ both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of $f$. Give a reason for your answer.

<table>
<thead>
<tr>
<th>If you want $f$ to be...</th>
<th>$f'$ should be...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ is increasing</td>
<td>$f'$ is positive</td>
</tr>
<tr>
<td>$f$ is decreasing</td>
<td>$f'$ is negative</td>
</tr>
<tr>
<td>$f$ has a relative max</td>
<td>$f'$ changes from positive to negative</td>
</tr>
<tr>
<td>$f$ has a relative min</td>
<td>$f'$ changes from negative to positive</td>
</tr>
<tr>
<td>$f$ is concave up</td>
<td>$f'' &gt; 0$</td>
</tr>
<tr>
<td>$f$ is concave down</td>
<td>$f'' &lt; 0$</td>
</tr>
<tr>
<td>$f$ has a point of inflection</td>
<td>$f''$ changes sign</td>
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</tbody>
</table>

- **$f''$ decreasing / $f''$ negative**
- **$f''$ changes sign**
- **Slope of $f'$ changes sign**

- **a)** $f$ has a relative max $\Rightarrow f'$ changes from positive to negative
- **b)** $(-2, -1)$ and $(1, 3)$ because $f'$ is decreasing and negative
- **c)** $f$ has points of inflection at $x = -1, 1, 3$ because the slope of $f'$ changes sign
The figure above shows the graph of the piecewise-linear function $f$. For $-4 \leq x \leq 12$, the function $g$ is defined by $g(x) = \int_{-4}^{x} f(t) \, dt$. This means $g'(x) = f(x)$.

(a) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of $g$ have a point of inflection at $x = 4$? Justify your answer.

- $g'$ changes sign
- Slope of $g'$ changes sign

a) Neither because $g'$ does not change sign
b) Yes because $g'$ has a relative max

$\rightarrow g'$ changes from increasing to decreasing
$\rightarrow$ slope of $g'$ changes sign
Use the graph below to answer the following. What do you notice about the concavity of the graph at the minimum and the maximum?

Let $f$ be a function such that $f'(c) = 0$ and the second derivative of $f$ exists on an open interval containing $c$.

1. If $f''(c) > 0$, then $f$ has a relative min at $x = c$.
2. If $f''(c) < 0$, then $f$ has a relative max at $x = c$.
3. If $f''(c) = 0$, then the 2nd deriv test fails. Use 1st deriv test w/ sign chart.

Find the relative extrema for $f(x) = -3x^5 + 5x^3$ using the second derivative test.

$f'(x) = -15x^4 + 15x^2$
$0 = -15x^2(x^2 - 1)$
$x = 0$ $x = \pm 1$

$f''(x) = -60x^3 + 30x$
$f''(0) = 0$
Test fails.

$f''(1) = -60 + 30 < 0$
$f$ has a relative max at $x = 1$ because $f'(1) = 0$ and $f''(1) < 0$

$f''(-1) = 60 - 30 > 0$
$f$ has a relative min at $x = -1$ because $f'(-1) = 0$ and $f''(-1) > 0$
Find all relative extrema of \( g(x) = x^3 - 3x^2 + 3 \). Start with Second Derivatives Test (check your answer using First Derivative Test).

2015 #4: Consider the differential equation \( \frac{dy}{dx} = 2x - y \).

Find \( \frac{d^2y}{dx^2} \) in terms of \( x \) and \( y \). Determine the concavity of all solutions for the given differential equation in Quadrant II. Give a reason for your answer.

Let \( y = f(x) \) be the particular solution to the differential equation with the initial condition \( f(2) = 4 \). Does \( f \) have a relative minimum, a relative maximum, or neither at \( x = 2 \)? Justify your answer.