Exploration: First Derivative Test

1. What are the critical numbers of the above function? How did you know that those were critical numbers?
   \[
   f' = 0 \quad \text{b, c, d, e, f, g}
   \]
   \[f' \text{ is undefined}\]

2. Fill out the chart:

<table>
<thead>
<tr>
<th>x-value Interval</th>
<th>a</th>
<th>(a, b)</th>
<th>b</th>
<th>(b, c)</th>
<th>c</th>
<th>(c, d)</th>
<th>d</th>
<th>(d, e)</th>
<th>e</th>
<th>(e, f)</th>
<th>f</th>
<th>(f, g)</th>
<th>g</th>
<th>(g, h)</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of (\frac{dy}{dx})</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>undef</td>
<td>-</td>
<td>undef</td>
<td>+</td>
<td>undef</td>
<td>undef</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Relative Max or min?</td>
<td>?</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td></td>
</tr>
</tbody>
</table>

3. Describe the behavior at and around a relative maximum.
   \[
   \frac{dy}{dx} \text{ changes from positive to negative}
   \]

4. Describe the behavior at and around a relative minimum.
   \[
   \frac{dy}{dx} \text{ changes from negative to positive}
   \]

5. Come up with a series of steps that will allow you to find where relative maxima and minima are of any given function (without a graph). Make your steps as specific and clear as possible. Be prepared to share with the class!
   1. Take a derivative
   2. Find critical #s \(f' = 0\) or undef
   3. Make sign chart
Notes: 5.3 Connecting $f'$ and $f''$ with the Graph of $f$

The First Derivative Test

- If $f'(x) > 0$, then $f$ is increasing on $[a, b]$.
- If $f'(x) < 0$, then $f$ is decreasing on $[a, b]$.
- If $f'(x) = 0$, then $f$ is constant on $[a, b]$.

The First Derivative Test

Let $c$ be a critical number of a function $f$ that is continuous on an open interval $I$ containing $c$. If $f$ is differentiable on the interval, except possibly at $c$, then $f(c)$ can be classified as follows.

- If $f'(x)$ changes from \underline{positive} to \underline{negative} at $c$ then $f$ has a relative \underline{maximum} at $(c, f(c))$.

- If $f'(x)$ changes from \underline{negative} to \underline{positive} at $c$ then $f$ has a relative \underline{minimum} at $(c, f(c))$.

- If $f'(x)$ is positive on both sides of $c$ or negative on both sides of $c$, then $(c, f(c))$ is neither a relative max nor a relative min.
Find the relative extrema of the function \( f(x) = \frac{1}{2}x - \sin x \) on \((0, 2\pi)\).

\[
f'(x) = \frac{1}{2} - \cos x
\]

- Critical Values:
  - \( x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3} \)

<table>
<thead>
<tr>
<th>Intervals</th>
<th>((0, \frac{\pi}{3}))</th>
<th>(\left(\frac{\pi}{3}, \frac{\pi}{2}\right))</th>
<th>(\left(\frac{\pi}{2}, \frac{\pi}{3}\right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of (f')</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Behavior of (f)</td>
<td>Decreasing</td>
<td>Increasing</td>
<td>Decreasing</td>
</tr>
</tbody>
</table>

- \( f(x) \) has a relative \text{ min} \ at \( x = \frac{\pi}{3} \) because \( f'(x) \) changes from \text{ negative} \ to \text{ positive}.
- \( f(x) \) has a relative \text{ max} \ at \( x = \frac{5\pi}{3} \) because \( f'(x) \) changes from \text{ positive} \ to \text{ negative}.

Find the relative extrema of \( f(x) = (x^2 - 4)^3 \).

<table>
<thead>
<tr>
<th>On what intervals is ( f(x) ) increasing?</th>
<th>On what intervals is ( f(x) ) decreasing?</th>
</tr>
</thead>
</table>
Find the relative extrema of \( f(x) = \frac{x^4 + 1}{x^2} \). 

The function \( f \) given by \( f(x) = x^3 + 12x - 24 \) is 
(A) increasing for \( x < -2 \), decreasing for \(-2 < x < 2\), increasing for \( x > 2 \) 
(B) decreasing for \( x < 0 \), increasing for \( x > 0 \) 
(C) increasing for all \( x \) 
(D) decreasing for all \( x \) 
(E) decreasing for \( x < -2 \), increasing for \(-2 < x < 2\), decreasing for \( x > 2 \) 

The function defined by \( f(x) = x^3 - 3x^2 \) for all real numbers \( x \) has a relative maximum at \( x = \) 
(A) \(-2\)  
(B) 0  
(C) 1  
(D) 2  
(E) 4