Free Response Practice

Consider the curve given by $x^2 - x^3 y = 0$.

1) **Show that** \( \frac{dy}{dx} \) **on one side to solve** (implicit \( \frac{dy}{dx} \))

   - \( \frac{dy}{dx} = \frac{3x^2 - 2x^3 y}{2x^3} \)
   - This is my answer
   - there is more than one

2) **Find all points on the curve whose x-coordinate is 1**, and write an equation for the tangent line at each of these points.

   - Plug in 1 into original
   - \( y - y_1 = m(x - x_1) \)
   - \( \frac{dy}{dx} \)

3) **Find the x-coordinate of each point on the curve where the tangent line is vertical**.

   - \( x = \frac{m}{x} = \frac{1}{x} \)
   - \( m = \text{undefined} \)
   - \( \text{denom} = 0 \)

   - \( 2xy - x^3 = 0 \)
   - \( 2y = x^3 \)
   - \( y = \frac{x^3}{2x} = \frac{x^2}{2} \)

   - \( xy^2 - x^3 y = 0 \)
   - \( x(\frac{x^2}{2}^2) - x^3 (\frac{x^2}{2}) = 0 \)
   - \( \frac{x^5}{4} - \frac{x^5}{2} = (0)4 \)
   - \( 1x^5 - 2x^5 = 24 \)
Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

a) Show that $\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$.

b) Show that there is a point $P$ with $x$-coordinate 3 at which the line tangent to the curve at $P$ is horizontal. Find the $y$-coordinate of $P$.

c) Find the value of $\frac{d^2y}{dx^2}$ at the point $P$ found in part (b).
Notes: 5.1 Extreme Values of Functions

TURN AND TALK: If you have a function defined on a given interval, what is its maximum? What is its minimum? Describe it with words.

- **maximum**: highest y-value
- **minimum**: lowest y-value

\( f(c) \) is an (absolute) minimum if \( f(c) \leq f(x) \) for all \( x \) in the interval.

\( f(c) \) is an (absolute) maximum if \( f(c) \geq f(x) \) for all \( x \) in the interval.

The maximums and minimums of a function on an interval are also called the **extreme values or extrema**.

TURN AND TALK: Try to draw a picture of a function that does not have a maximum on a given interval.

How can you GUARANTEE that a function has both a minimum and a maximum on a given interval?

Function is continuous interval is closed (include the ends)

The Extreme Value Theorem:

If \( f \) is continuous on the closed interval \([a, b]\) then \( f \) has a maximum and a minimum on \([a, b]\)

\( f(c) \) is a relative/local maximum if there is an open interval where \( f(c) \) is a max.

\( f(c) \) is a relative/local minimum if there is an open interval where \( f(c) \) is a min.

If it's just the biggest/smallest number on the interval, it's called **absolute**.

If \( f'(c) = 0 \) or \( f'(c) \) is undefined, then \( c \) is a critical number of \( f \).
Finding absolute extrema

**abs max**  
\( (2, 16) \)

**abs min**  
\( (1, -1) \)

**min value**  
\(-7\)

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Find the absolute extrema of \( f(x) = 3x^4 - 4x^3 \) on the interval \([-1, 2]\).

\[ f'(x) = 12x^3 - 12x^2 \]

- \( x = 0 \)  
  \( f(0) = 0 \)
  \( f'(x) = 3x^2 - 4x \)
  \( 1 - 4 = -3 \)
  \( x = 1 \)
  \( f(1) = 3 + 4 = 7 \)

- \( x = 1 \)  
  \( f(1) = 7 \)

Let \( h(x) = 2x^3 + 3x^2 - 12x \). What is the minimum value of \( h \) over the closed interval \(-3 \leq x \leq 3\)?

\[ h'(x) = 6x^2 + 6x - 12 \]

- \( x = 2 \)  
  \( h(2) = 20 \)
  \( y \)-value

- \( x = 1 \)  
  \( h(1) = 7 \)

If \( g(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5 \) and the domain is the set of all \( x \) such that \( 0 \leq x \leq 9 \), then the absolute maximum value of the function \( f \) occurs when \( x \) is

- a. 0
- b. 2
- c. 4
- d. 6
- e. 9

The maximum acceleration attained on the interval \( 0 \leq t \leq 3 \) by the particle whose velocity is given by \( v(t) = t^3 - 3t^2 + 12t + 4 \) is

- a. 9
- b. 12
- c. 14
- d. 21
- e. 40

What is the minimum value of \( f(x) = x \ln x \)?

- a. \(-e\)
- b. \(-1\)
- c. \(-\frac{1}{e}\)
- d. 0
- e. \( f(x) \) has no minimum

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