1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

\[ \frac{dr}{dt} = 4 \]

\[ A = \pi r^2 \]

\[ \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \]

\[ \frac{dA}{dt} = 2\pi (5)(4) \]

\[ = 40\pi \frac{cm^2}{min} \]
6) An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?

\[ c = 2500 \]
\[ \frac{dc}{dt} = ? \]
\[ b = 2400 \]
\[ \frac{db}{dt} = 900 \]
\[ a = 700 \]
\[ \frac{da}{dt} = 0 \]

\[ a^2 + b^2 = c^2 \]
\[ 2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt} \]
\[ 700(0) + 2400(900) = 2500 \frac{dc}{dt} \]
\[ \frac{dc}{dt} = 864 \text{ ft/sec} \]
5) A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

\[
\frac{7}{20} = \frac{s}{a+s}
\]

\[
7a + 7s = 20s
\]

\[
7a = 13s
\]

\[
7\frac{da}{dt} = 13\frac{ds}{dt}
\]

\[
7(5) = 13\frac{ds}{dt}
\]

\[
\frac{ds}{dt} = \frac{35}{13} \quad \text{ft/sec}
\]

\[
= 2.692 \quad \text{ft/sec}
\]
### Notes: 4.5 Related Rates (with Substitution)

#### Review: The "classic" related rates example

The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

- a) $\frac{7}{6}$ feet per minute
- b) $\frac{7}{24}$ feet per minute
- c) $\frac{7}{24}$ feet per minute (d) $\frac{7}{8}$ feet per minute
- e) $\frac{21}{25}$ feet per minute

### Variable vs. Constant Rate

A cylindrical tank with radius 5 feet and height 15 feet that is filled with water has sprung a leak (oh no!). Water is leaving the tank at a rate of 8 cubic feet per minute. What is the rate of change of the height of the water when the water is 10 feet deep?

The volume $V$ of a cylinder with radius $r$ and height $h$ is: $V = \pi r^2 h$

#### Option 1

$V = \pi r^2 h$

\[
\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}
\]

\[
-8 = \pi (5)^2 \frac{dh}{dt} + 10(2\pi)(5)(10)
\]

\[
\frac{dh}{dt} = \frac{-8}{25\pi} \text{ ft/min}
\]
Sally places a marshmallow in the microwave to make a s’more. When she turns on the microwave, the marshmallow starts to expand at the rate of 2 cm³ per second. Assume also that as it grows, the height is always three times the radius. Find the rate at which the radius is increasing when the radius is 4 cm.

\[ h = 3r \]

\[
\begin{align*}
V &= \pi r^2 h \\
V &= \pi r^2 (3r) \\
V &= 3\pi r^3 \\
\frac{dV}{dt} &= 9\pi r^2 \frac{dr}{dt} \\
2 &= 9\pi (4)^2 \frac{dr}{dt} \\
\frac{dr}{dt} &= \frac{2}{144\pi} = 0.00444 \text{ cm/sec}
\end{align*}
\]

A boy launches a toy rocket 15 feet away with a remote control. The toy rocket gains altitude at a rate of 2.5 feet per second. Find the rate at which the angle of elevation is changing after 6 seconds.

\[ \tan \theta = \frac{h}{15} \]

\[ \tan \theta = \frac{15}{15} h \]

\[ \sec^2 \theta \frac{d\theta}{dt} = \frac{1}{15} \frac{dh}{dt} \]

\[
\begin{align*}
\frac{d\theta}{dt} &= \frac{1}{15} \frac{dh}{dt} \cos^2 \theta \\
\frac{d\theta}{dt} &= \frac{1}{15} (2.5) \left( \frac{15}{15\sqrt{2}} \right)^2 \\
&= \frac{1}{15} (2.5) \left( \frac{1}{2} \right) \\
&= 0.083 \text{ rad/sec}
\end{align*}
\]
A conical tank (with vertex down) is 4 feet across the top and 6 feet deep. If water is flowing into the tank at a rate of 3 cubic feet per minute, find the rate of change of the depth of the water when the water is 3 feet deep.

The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height $h$, the radius of the funnel is given by $r = \frac{1}{20} (3 + h^2)$, where $0 \leq h \leq 10$. The units of $r$ and $h$ are inches.

c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{2}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?