DO NOW

Draw the derivative of the following functions:

Draw the derivative of the following function (do one piece at a time)
Notes: 3.4 Quotient Rule

Use the quotient rule and \( \tan x = \frac{\sin x}{\cos x} \) to find

\[
\frac{d}{dx} \left[ \tan x \right] = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}
\]

\[
= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}
\]

\[
= \frac{1}{\cos^2 x} = \sec^2 x
\]

Use the quotient rule and \( \sec x = \frac{1}{\cos x} \) to find

\[
\frac{d}{dx} \left[ \sec x \right] = \frac{\cos x(\sec x) - \sec x(-\sin x)}{\cos^2 x}
\]

\[
= \frac{\cos x \cdot \sec x + \sec x \cdot \sin x}{\cos^2 x}
\]

\[
= \frac{\sec x(\sec x + \tan x)}{\cos^2 x}
\]

Use the quotient rule and \( \cot x = \frac{\cos x}{\sin x} \) to find

\[
\frac{d}{dx} \left[ \cot x \right] = \frac{\sin x(-\sin x) - \cos x \cdot \cos x}{\sin^2 x}
\]

\[
= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}
\]

\[
= \frac{-1}{\sin^2 x} = -\csc^2 x
\]

Use the quotient rule and \( \csc x = \frac{1}{\sin x} \) to find

\[
\frac{d}{dx} \left[ \csc x \right] = \frac{-\sin x(-\sin x) - \cos x \cdot \cos x}{\sin^2 x}
\]

\[
= \frac{-\sin x \cdot \csc x - \cos x}{\sin^2 x}
\]

\[
= \frac{-\csc x \cdot \cot x}{\sin^2 x}
\]
Chapter 3 Derivative Rules:

\[
\frac{d}{dx}[c] = 0 \\
\frac{d}{dx}[x^n] = nx^{n-1} \\
\frac{d}{dx}[e^x] = \text{next unit!} \\
\frac{d}{dx}[\ln x] = \text{next unit!} \\
\frac{d}{dx}[\sin x] = \cos x \\
\frac{d}{dx}[\cos x] = -\sin x \\
\frac{d}{dx}[\tan x] = \sec^2 x \\
\frac{d}{dx}[\csc x] = -\csc x \cot x \\
\frac{d}{dx}[\sec x] = \sec x \tan x \\
\frac{d}{dx}[\cot x] = -\csc^2 x \\
\frac{d}{dx}[f(x) \cdot g(x)] = f(x)g'(x) + g(x)f'(x) \\
\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}
\]
# Notes: 3.5 Position, Velocity, and Acceleration

<table>
<thead>
<tr>
<th>Particle Motion – Motion Along a Line</th>
<th>Position: $x(t)$ or $s(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position, Velocity, Acceleration (PVA)</td>
<td>Displacement: $x(b) - x(a)$ end position – start position</td>
</tr>
<tr>
<td></td>
<td>Average Velocity: $\frac{x(b) - x(a)}{b-a}$</td>
</tr>
<tr>
<td></td>
<td>Instantaneous Velocity: $v(t) = x'(t)$</td>
</tr>
<tr>
<td></td>
<td>Speed: $</td>
</tr>
<tr>
<td></td>
<td>Acceleration: $a(t) = v'(t) = x''(t)$ (instantaneous)</td>
</tr>
</tbody>
</table>

### Example: Function

A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where $s$ is measured in meters and $t$ is measured in seconds.

- **a)** Find the displacement of the particle during the first 2 seconds.
  
  $s(2) - s(0) = (4 - 8 + 3) - (0 - 0 + 3) = -4 \text{ m}$

- **b)** Find the average velocity of the particle during the first 4 seconds.
  
  $\frac{s(4) - s(0)}{4 - 0} = \frac{(16 - 16 + 3) - (0 - 0 + 3)}{4} = 0 \text{ m/sec}$

- **c)** Find the instantaneous velocity of the particle when $t = 4$.
  
  $v(t) = x'(t) = 2t - 4$
  
  $v(4) = 2(4) - 4 = 4 \text{ m/sec}$

- **d)** Find the acceleration of the particle when $t = 4$.
  
  $a(t) = v'(t) = 2$
  
  $a(4) = 2 \text{ m/s}^2$

- **e)** Determine the time of the particle. At what values of $t$ does the particle change direction? (particle is at rest)
  
  $v(t) = 0$
  
  $2t - 4 = 0$
  
  $t = 2 \text{ sec}$
1. What function is graphed above? **VELOCITY**

2. When is the function moving to the right? Explain your reasoning.
   - **Particle**
   - \((4, 6)\)
   - \(v(t) > 0\)

3. When is the function moving to the left? Explain your reasoning.
   - **Particle**
   - \([0, 4)\)
   - \(v(t) < 0\)

4. When is the velocity of the particle increasing?
   - \((2, 6)\)

5. When is the velocity of the particle decreasing?
   - \((0, 2)\)

6. When is the particle speeding up?
   - \((0, 2)\) \((4, 6)\)
   - **Graph goes up**
   - Away from \(v = 0\)
   - Towards \(v = 0\)

7. When is the particle slowing down?
   - \((2, 4)\)

8. How should we describe why we know the particle is speeding up or slowing down on the AP test?
   - \(v\) & \(a\) have different signs
   - \(v\) & \(a\) have same sign

9. Sketch the graph of the acceleration of the particle on the velocity-time graph above.
Today: 3.5 Position, Velocity, & Acceleration

Thursday: short review, 4.1 Chain rule, R2 Problem Set due!!!

Monday: NO VAHSEN- review unit 3 (test review, finish problem sets, correct notes checks)

Wednesday: NO VAHSEN- unit 3 test (round 1), Graded Problem Set due!!!