Notes: 3.3 Product Rule

Simple Trig Derivatives

| \( \frac{d}{dx} [\sin x] = \cos x \) | \( \frac{d}{dx} [\cos x] = -\sin x \) |

Find the derivative of \( f(x) = (x^3)(\sqrt{x}) \). Simplify first.

Is this the same as \( \frac{d}{dx} [x^3] \cdot \frac{d}{dx} [\sqrt{x}] \)?

Product Rule

The product of two differentiable functions \( f \) and \( g \) is itself differentiable. Moreover, the derivative of \( fg \) is the first function times the derivative of the second, plus the second function times the derivative of the first.

\[
\frac{d}{dx} [fg] = f(x)g'(x) + g(x)f'(x)
\]

Find the derivative of \( g(x) = (3x - 2x^2)(5 + 4x) \)

\[
g'(x) = (3x - 2x^2)(4) + (5 + 4x)(3 - 4x) \]

\[
g'(x) = 12x - 8x^2 + 15 - 20x + 12x - 16x^2
\]

\[
g'(x) = -24x^2 + 4x + 15
\]

Option #2: \( g(x) = 15x + 12x^2 - 10x^2 - 8x^3 \)

\[
g(x) = -8x^3 + 2x^2 + 15x
\]

\[
g'(x) = -24x^2 + 4x + 15
\]

TURN AND TALK: How else could I find the derivative of \( g \)?
Using the Product Rule

Let \( y = uv \) be the product of the functions \( u \) and \( v \). Find \( y'(2) \) if

\[
\begin{align*}
  u(2) &= 3 & u'(2) &= -4 & v(2) &= 1 & v'(2) &= 2
\end{align*}
\]

Find the slope of the line tangent to \( f(x) = 3x^2 \sin x \) for any value of \( x \).

Find the derivative of \( h(x) = 2x \cos x - 2 \sin x \).

\[
\begin{align*}
  h'(x) &= 2x(-\sin x) + \cos x \cdot 2 - 2 \cos x \\
  h'(x) &= -2x \sin x
\end{align*}
\]

The Second Derivative

Alternate Notation:

Newton

\( f''(x) \)

double prime

Leibniz

\[
\frac{d^2y}{dx^2} = \frac{d}{dx} \left[ \frac{dy}{dx} \right]
\]

Find \( \frac{d^2y}{dx^2} \) given \( y = x \cos x \).

\[
\begin{align*}
  \frac{dy}{dx} &= x(-\sin x) + \cos x \\
  &= -x \sin x + \cos x \\
  \frac{d^2y}{dx^2} &= -x(\cos x) + \sin x(-1) + (-\sin x) \\
  &= -x \cos x - 2 \sin x
\end{align*}
\]
Notes: 3.4 Quotient Rule

**Higher Order Derivative Notation**

original function is \( y: y', y'', y''', y^{(4)}, \ldots, y^{(n)} \), ...

original function is \( f(x): f'(x), f''(x), f'''(x), f^{(4)}(x), \ldots \)

original function is \( y: \frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots \)

Given \( f(x) = \cos x \), find \( f^{(4)}(x) \).

\[ f'(x) = -\sin x \]
\[ f''(x) = -\cos x \]
\[ f'''(x) = \sin x \]
\[ f^{(4)}(x) = \cos x \]

Find the derivative of the function \( f(x) = \frac{3x^3 - 4x + 3}{x} \) by rewriting first.

\[ f(x) = \frac{3x^2}{x} - \frac{4x}{x} + \frac{3}{x} = 3X - 4 + 3X^{-1} \]

\[ f'(x) = 3 - 3x^{-2} = 3 - \frac{3}{x^2} \]

Is this the same as \( \frac{d}{dx} \left[ \frac{3x^3 - 4x + 3}{x} \right] \)?

\[ \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \]

\[ \frac{d}{dx} \left( \frac{u}{v} \right) = \frac{vdu - udv}{v^2} \]

\[ \frac{d}{dx} \left( \frac{hi}{lo} \right) = \frac{lo \cdot d(hi) - hi \cdot d(lo)}{[lo]^2} \]

\[ \frac{d}{dx} \left( \frac{hi}{ho} \right) = \frac{ho \cdot d(hi) - hi \cdot d(ho)}{ho \cdot ho} \]
Using the Quotient Rule

Find the derivative of \( f(x) = \frac{5x-2}{x^2+1} \)

\[
f'(x) = \frac{(x^2+1)(5) - (5x-2)(2x)}{(x^2+1)^2}
\]

\[
f'(x) = \frac{5x^2+5 - 10x^2 + 4x}{(x^2+1)^2} = \frac{-5x^2 + 4x + 5}{(x^2+1)^2}
\]

Find the derivative of \( g(x) = \frac{3}{x-5} \) (HINT: Simplify first!). Then find the slope of the normal line at \((-1, 1)\).

\[
g(x) = \frac{3-x}{x-5} \implies g(x) = \frac{3x-1}{x^2-5x}
\]

\[
g'(x) = \frac{(x^2-5x)(3) - (3x-1)(2x-5)}{(x^2-5x)^2} = \frac{18-28}{36} = \frac{-10}{36} = \frac{-5}{18}
\]

Find equations of the tangent lines to the graph of \( f(x) = \frac{x+1}{x-1} \) that are parallel to the line \( 2y+x=6 \).

\[
f'(x) = \frac{(x-1)(1)-(x+1)(1)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}
\]

\[
f'(x) = \frac{1}{2} = -\frac{2}{(x-1)^2}
\]

\[
2y+x = 6 \implies y = -\frac{1}{2}x + 3 \implies m = -\frac{1}{2}
\]

Find the derivative of \( y = \frac{x^2+3}{2x} \)

\[
\frac{dy}{dx} = \frac{(2x)(x^2+3) - (x^2+3)(2x)}{(2x)^2}
\]

\[
= \frac{2x^3 + 6x - 2x^3 - 6x}{4x^2} = 0
\]

\[
f(-1) = 0
\]

\[
f(3) = \frac{4}{2} = 2
\]

Is the Quotient Rule always needed for rational functions?
next class: partner quiz

• power rule
• product rule
• quotient rule
• derivs of sinx & cosx