DO NOW – Chapter 2 Review

1. Find the value of \( k \) such that \( f(x) \) is continuous at \( x = 2 \).

\[
f(x) = \begin{cases} 
3x & \text{if } x \leq 2 \\
\frac{3x}{kx^2 - 6} & \text{if } x > 2
\end{cases}
\]

\[
3(2) = k(2)^2 - 6
\]

\[12 = 4k - 6\]

\[k = 3\]

2. Given \( f(x) = 3x^2 + 4x + 7 \).

a. Find the slope of the line tangent to graph of \( f(x) \) for any value of \( x \).

\[
\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{3(x + \Delta x)^2 + 4(x + \Delta x) + 7 - (3x^2 + 4x + 7)}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{3x^2 + 6x\Delta x + 3\Delta x^2 + 4x + 4\Delta x + 7 - 3x^2 - 4x - 7}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{6x + 3\Delta x + 4}{\Delta x}
\]

\[= 6x + 4\]

b. Use your answer to part a to find where the slope of the line tangent to \( f(x) \) is 18.

\[6x + 4 = 18\]

\[6x = 14\]

\[x = \frac{7}{3}\]

c. Use your answer to part a to find an equation of the line tangent to the curve at \( x = -1 \).

\[m = 6(-1) + 4 = -2\]

\[y = f(-1) = 3(-1)^2 + 4(-1) + 7\]

\[= 3 - 4 + 7\]

\[= 6\]

\[y - y_1 = m(x - x_1)\]

\[y - 6 = -2(x + 1)\]
Notes: 3.1 Derivatives & Differentiability

<table>
<thead>
<tr>
<th>Definition of the DERIVATIVE of a function</th>
<th>The derivative of the function ( f ) with respect to the variable ( x ) is given by...</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} ]</td>
<td></td>
</tr>
<tr>
<td>[ = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} ]</td>
<td></td>
</tr>
<tr>
<td>provided the limit exists. For all ( x ) for which this limit exists, ( f' ) is a function of ( x ).</td>
<td></td>
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</tbody>
</table>

### NOTATION

**Gottfried Leibniz**

### Using the limit definition of the derivative

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Pronunciation</th>
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<tr>
<td>( f'(x) )</td>
<td>( f ) prime of ( x )</td>
</tr>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>( d ) ( y ) ( d ) ( x ) (derivative of ( y ) with respect to ( x ))</td>
</tr>
<tr>
<td>( \frac{d}{dx}[f(x)] )</td>
<td>derivative of ( f(x) ) with respect to ( x )</td>
</tr>
</tbody>
</table>

Find the derivative of \( f(x) = x^4 \):

\[
f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3-x^3}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{|x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 - x^3|}{\Delta x}
\]

\[
= \lim_{\Delta x \to 0} \frac{\Delta x(x^2 + 3x \Delta x + \Delta x^2)}{\Delta x}
\]

\[
f'(x) = 3x^2
\]

### What is a derivative?

**TURN AND TALK:** In five words or less, what is a derivative?

**slope of the tangent line**

**IF I SAY DERIVATIVE, YOU SAY **SLOPE**!**
Alternate form of the derivative:

\[ \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \]

Using the alternative limit form of the derivative, find the slope of the graph of \( f(x) = \sqrt{x} \) at \((4, 2)\).

\[
\lim_{x \to 4} \frac{\sqrt{x} - \sqrt{4}}{x - 4}
\]

\[
\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}
\]

\[
\lim_{x \to 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)}
\]

\[
1 \quad (\sqrt{4} + 2) = \frac{1}{4}
\]

Use the point-slope equation for a line:

\[
y - y_1 = m(x - x_1)
\]

Using the derivative you found in the example above, find the equation of the tangent line to the curve \( f(x) = \sqrt{x} \) at \((4, 2)\).

\[
y - 2 = \frac{1}{4}(x - 4)
\]
**Important Vocab**

<table>
<thead>
<tr>
<th>Differentiation:</th>
<th>the process of taking derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differentiable:</td>
<td>able to take a derivative</td>
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</table>

**How to tell if something is differentiable**

**HARD WAY:** Use a limit definition of the derivative to find derivatives from both sides (alternate)

**EASY WAY:** Lance Armstrong!

**Three main types of functions that are NOT differentiable.**

<table>
<thead>
<tr>
<th>TYPE 1:</th>
<th>not continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: $f(x) = [x]$</td>
<td></td>
</tr>
<tr>
<td>$f(2) = [2] = 2$</td>
<td></td>
</tr>
<tr>
<td>$f(2.3) = [2.3] = 2$</td>
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</table>

<table>
<thead>
<tr>
<th>TYPE 2:</th>
<th>sharp turn</th>
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</thead>
<tbody>
<tr>
<td>Example: $f(x) =</td>
<td>x - 2</td>
</tr>
<tr>
<td>Why not? $\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = -1$</td>
<td></td>
</tr>
<tr>
<td>$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = 1$</td>
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Differentiability and Continuity

TURN AND TALK: How do I find out if a function is differentiable at a x-value, c?

not not differentiable
not 3 examples

\[ \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \infty \]

or DNE

TURN AND TALK: If \( f \) is continuous at \( c \) then \( f \) is differentiable at \( c \)? Explain why or draw a counterexample:

sharp turn
Unit 2 Test Talk

Content:
- limits graphically (find limit, draw a function)
- limits analytically (direct sub, factor, conjugates, common denom, expand, special trig)
- limits w/ infinity (HA rules, checking signs to choose)
- continuity (proof, find the constant)
- IVT
- average rate of change
- slope of tangent line (delta x definition, find where slope is something, write a tangent line)
Unit 2 Test Talk

Format:
- 22 questions (some short answer, some multiple choice)
- partial credit on everything (show work on multiple choice)
- no calculator
- bonus question (hint: one-sided limits)
- reminder: phones up at board for test
Unit 2 Test Talk

Timing:
- Round 1 is NEXT CLASS (Wednesday)
- Round 2 will most likely be on Friday
- full class to take test
- Graded Problem Set also due NEXT CLASS (Wednesday)
- R2 prob set corrections by Wednesday