1. Find the value of $k$ such that $f(x)$ is continuous at $x = 2$.

$$f(x) = \begin{cases} 3x & x \leq 2 \\ \frac{3x}{kx^2 - 6} & x > 2 \end{cases}$$

$$3(2) = k(2)^2 - 6$$

$$12 = 4k - 6$$

$$k = 3$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

2. Given $f(x) = 3x^2 + 4x + 7$.
   a. Find the slope of the line tangent to the graph of $f(x)$ for any value of $x$.

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{(3(x + \Delta x)^2 + 4(x + \Delta x) + 7) - (3x^2 + 4x + 7)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{3x^2 + 6x \Delta x + 3 \Delta x^2 + 4x + 4 \Delta x + 7 - 3x^2 - 4x - 7}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x(6x + 3\Delta x + 4)}{\Delta x} = 6x + 4$$

b. Use your answer to part a to find where the slope of the line tangent to $f(x)$ is 18.

$$6x + 4 = 18$$

$$6x = 14$$

$$x = \frac{14}{6}$$

$$x = \frac{7}{3}$$

c. Use your answer to part a to find an equation of the line tangent to the curve at $x = -1$.

$$m = 6(-1) + 4 = -2$$

$$f(-1) = 3(-1)^2 + 4(-1) + 7 = 3 - 4 + 7 = 6$$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -2(x + 1)$$
Notes: 3.1 Derivatives & Differentiability

The derivative of the function \( f \) with respect to the variable \( x \) is given by...

\[
 f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

\[
 = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

provided the limit exists. For all \( x \) for which this limit exists, \( f' \) is a function of \( x \).

Different ways to write the derivative:

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Pronunciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td>\textit{f prime of } x</td>
</tr>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>( \text{dy dx (derivative of } y \text{ with respect to } x )</td>
</tr>
<tr>
<td>( \frac{d}{dx}[f(x)] )</td>
<td>derivative of ( f(x) ) with respect to ( x )</td>
</tr>
</tbody>
</table>

Using the limit definition of the derivative

Find the derivative of \( f(x) = x^3 \):

\[
 f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}
\]

\[
 = \lim_{\Delta x \to 0} \frac{x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 - x^3}{\Delta x}
\]

\[
 = \lim_{\Delta x \to 0} \frac{\Delta x(3x^2 + 3x \Delta x + \Delta x^2)}{\Delta x}
\]

\[
 f'(x) = 3x^2
\]

What is a derivative?

TURN AND TALK: In five words or less, what is a derivative?

\textit{slope of tangent line}

IF I SAY DERIVATIVE, YOU SAY \textsc{slope} !
Alternate form of the derivative:
\[ \lim_{{x \to c}} \frac{f(x) - f(c)}{x - c} \]

Using the alternative limit form of the derivative, find the slope of the graph of \( f(x) = \sqrt{x} \) at (4, 2).

\[
\lim_{{x \to 4}} \frac{\sqrt{x} - \sqrt{4}}{x - 4} = \frac{1}{2} \\
\lim_{{x \to 4}} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \frac{1}{2} \\
\lim_{{x \to 4}} \frac{x - 4}{(x-4)(\sqrt{x}+2)} = \frac{1}{2} \\
\frac{1}{\sqrt{4} + 2} = \frac{1}{2}
\]

Use the point-slope equation for a line:
\[ y - y_1 = m(x - x_1) \]

Using the derivative you found in the example above, find the equation of the tangent line to the curve \( f(x) = \sqrt{x} \) at (4, 2).

\[ y - 2 = \frac{1}{4}(x - 4) \]
<table>
<thead>
<tr>
<th>Important Vocab</th>
<th>Differentiation: <strong>the process of taking derivatives</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Differentiable: <strong>able to take a derivative</strong></td>
</tr>
<tr>
<td></td>
<td>HARD WAY: Use a limit definition of the derivative to <strong>find a derivative from both sides (alternate)</strong></td>
</tr>
<tr>
<td></td>
<td>EASY WAY: Lance Armstrong!</td>
</tr>
</tbody>
</table>

There are three main types of functions that are NOT differentiable.

**TYPE 1:** not continuous

Example: $f(x) = \lfloor x \rfloor$ — greatest integer function

- $f(2) = \lfloor 2 \rfloor = 2$
- $f(2.3) = \lfloor 2.3 \rfloor = 2$

**TYPE 2:** sharp turn

Example: $f(x) = |x - 2|$

Why not?

\[
\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = -1
\]

\[
\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = 1
\]
TYPE 3: **vertical tangent line**

Example: \( f(x) = x^{\frac{1}{3}} \)

Why not?

\[
\lim_{{x \to 0}} \frac{f(x) - f(0)}{x - 0} = \infty \quad \text{(or DNE)}
\]

**TURN AND TALK:** How do I find out if a function is differentiable at a \( x \)-value, \( c \)?

- not differentiable
- not one of 3 examples

**Differentiability implies continuity**

If \( f \) is differentiable at \( x = c \), then \( f \) is continuous at \( x = c \).

**TURN AND TALK:** If \( f \) is continuous at \( c \) then \( f \) is differentiable at \( c \)? Explain why or draw a counterexample:

---

**Sharp turn**
Unit 2 Test Talk

Content:
- limits graphically (find limit, draw a function)
- limits analytically (direct sub, factor, conjugates, common denom, expand, special trig)
- limits w/ infinity (HA rules, checking signs to choose)
- continuity (proof, find the constant)
- IVT
- average rate of change
- slope of tangent line (delta x definition, find where slope is something, write a tangent line)
Unit 2 Test Talk

Format:
- 22 questions (some short answer, some multiple choice)
- partial credit on everything (show work on multiple choice)
- no calculator
- bonus question (hint: one-sided limits)
- reminder: phones up at board for test
Unit 2 Test Talk

Timing:
- Round 1 is NEXT CLASS (Wednesday)
- Round 2 will most likely be on Friday
- full class to take test
- Graded Problem Set also due NEXT CLASS (Wednesday)
- R2 prob set corrections by Wednesday