To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the FlexBook®, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the FlexBook Platform®.

Copyright © 2012 CK-12 Foundation, www.ck12.org

The names “CK-12” and “CK12” and associated logos and the terms “FlexBook®” and “FlexBook Platform®” (collectively “CK-12 Marks”) are trademarks and service marks of CK-12 Foundation and are protected by federal, state, and international laws.

Any form of reproduction of this book in any format or medium, in whole or in sections must include the referral attribution link http://www.ck12.org/saythanks (placed in a visible location) in addition to the following terms.

Except as otherwise noted, all CK-12 Content (including CK-12 Curriculum Material) is made available to Users in accordance with the Creative Commons Attribution/Non-Commercial/Share Alike 3.0 Unported (CC BY-NC-SA) License (http://creativecommons.org/licenses/by-nc-sa/3.0/), as amended and updated by Creative Commons from time to time (the “CC License”), which is incorporated herein by this reference.

Complete terms can be found at http://www.ck12.org/terms.

Printed: November 14, 2012
Introduction

The Hot Dogs

One day after diving, Cameron and some of the kids that he has met at the resort decide to eat some hotdogs on the beach. Cameron takes everyone’s order and heads to the hot dog stand. He figures that he will get a tray. Each of the kids has given him a few dollars and Cameron thinks that he has enough money to get everything.

When he gets to the stand, he checks the prices. Cameron needs to buy 9 hotdogs. The hotdogs are $1.50 for a plain dog plus $1.00 for cheese and sauce. Everyone wants cheese and sauce, so Cameron needs to buy nine hotdogs with cheese and sauce.

Given these numbers, how much will Cameron spend?

Cameron isn’t sure. He takes a napkin and asks for a pen so that he can figure it all out. He has $25.00. Does he have enough?

The Distributive Property will be very helpful to Cameron.

**What is the Distributive Property?** Well, this is the lesson that will teach you all about it. Pay attention and at the end of this lesson you will help Cameron buy some lunch.

**What You Will Learn**

In this lesson, you will learn the following skills.

- Identify the Distributive Property of Multiplication over Addition.
- Use the Distributive Property to write equivalent numerical and algebraic expressions.
- Use the Distributive Property to evaluate numerical and algebraic expressions.
- Model and solve real-world problems using simple equations involving rational numbers.

**Teaching Time**

1. **Identify the Distributive Property of Multiplication over Addition**
You have just finished working with several different properties and are probably beginning to develop a sense of how useful properties are when working with numbers. However, there is another property that you haven’t learned yet. This is a property that combines multiplication and addition. It is called the **Distributive Property of Multiplication**. Often, you will hear it referred to simply as “The Distributive Property.”

The Distributive Property states that when a factor is multiplied by the sum of two numbers, we can multiply each of the two numbers by that factor and then perform the operation indicated within the parentheses. When the Distributive Property is used, you will see a term outside of the parentheses and then you will know that we are dealing with the Distributive Property.

Example

\[
6(3 + 5) \\
5(x + 3)
\]

Both of these are examples where we would utilize the Distributive Property. You can see that the operation outside the parentheses is multiplication and the operation inside the parentheses is addition. You can also see the Distributive Property of Multiplication being used when numbers within the parentheses are being subtracted.

Example

\[
5(7 - 4) \\
3(x - 2)
\]

Both of these examples would require the use of the Distributive Property.

II. **Use the Distributive Property to Write Equivalent Numerical and Algebraic Expressions**

You can use the Distributive Property to write equivalent expressions. You know that equivalent means equal. Well, we can write equivalent numerical and algebraic expressions using the Distributive Property.

**How do we write an equivalent numerical expression?**

You write an equivalent numerical expression by writing the expression without the parentheses. To do this, we multiply the term outside the parentheses with both terms inside the parentheses.

Example

\[
5(2 + 3)
\]
Here we multiply five times 2 and five times 3. The addition sign stays between the two terms.

\[ 5(2 + 3) = 5(2) + 5(3) \]

This is an equivalent numerical expression.

How do we write equivalent algebraic expressions?

Well, an algebraic expression is going to involve numbers, operations, variables and sometimes exponents too. We simply take the term outside the parentheses and multiply it with both of the terms inside the parentheses.

Example

\[ 4(x + 3) \]

Here we multiply four times \( x \) and four times 3. The addition sign stays in the middle.

\[ 4(x + 3) = 4(x) + 4(3) \]

This is an equivalent algebraic expression.

Take a few minutes to write down the steps in using the Distributive Property.

4P. Lesson Exercises

Write an equivalent expression for each using the Distributive Property.

a. \( 6(5 + 2) \)
b. \( 3(x - 5) \)
c. \( 8(9 + y) \)

Take a few minutes to check your work with a partner.

III. Use the Distributive Property to Evaluate Numerical and Algebraic Expressions

You can also use the Distributive Property to evaluate an expression. You might have caught yourself trying to do that in the last section. Well, the first step is to write an equivalent expression as we did in the last section and then we can simplify our work.

Let’s start with a numerical expression. Because a numerical expression does not contain a variable, we will be able to figure out an answer for the expression.

Example
7(2 + 3)
First, we write an equivalent expression.
7(2 + 3) = 7(2) + 7(3)
Next, we multiply each part and then we add the products.
14 + 21
Our answer is 35.
This works the same way if there was subtraction involved.
Example
3(3 − 2)
First, we write an equivalent expression.
3(3) − 3(2)
Next, we evaluate the expression.
9 − 6
Our answer is 3.
How does this work with an algebraic expression?
We can follow the same procedure, but keep in mind that an algebraic expression will have variables in it. Therefore, we can simplify the expression, but not necessarily solve it.
Example
2(x + 6)
First, we write an equivalent expression.
2(x) + 2(6)
Next, we simplify each part of the expression.
2x + 12
This is our answer.
Example
5(y − 2)
First, we write an equivalent expression.
5(y) − 5(2)
Next, we simplify each part of the expression.
5y − 10
This is our answer.
Sometimes, we can use the Distributive Property to make it easier to evaluate an expression. It can sometimes even allow us to use mental math to quickly compute an answer.
Example
Evaluate this expression 6 × 209.
This math can be done mentally if we think of 209 as a sum.
6 × 209 = 6 × (200 + 9)
Now, we can see how the Distributive Property would allow us to compute the answer mentally.

\[ 6 \times (200 + 9) = (6 \times 200) + (6 \times 9) = 1,200 + 54 = 1,254 \]

So, the value of the expression is 1,254.

4Q. Lesson Exercises

Practice evaluating these expressions.

a. \( 4(2 + 7) \)

b. \( 6(x - 2) \)

c. \( 12(y + 4) \)

Take a few minutes to check your work with a friend. Is your work accurate?

IV. Model and Solve Real-World Problems Using Simple Equations Involving Rational Numbers

Next, we can take what we have learned and apply it to our work with some real-world problems.

Example

Liam has a rectangular backyard that is 20 yards long and 18 yards wide. He wants to use a part of his yard that is 20 yards by 8 yards for a vegetable garden. If he does this, what will be the area of the section of the yard that will not be used as a garden?

Let’s start by drawing a diagram of Liam’s backyard to help us understand this problem better.

One way we can find the area of the section that will not be used as a garden is by subtracting the area of the garden from the total area of the yard.
Remember, to find the area of any rectangle, including a rectangular yard, multiply the length times the width.

\[(\text{area of entire yard}) - (\text{area of garden}) = (\text{area of section not used as a garden})\]

\[(20 \times 18) - (20 \times 8) = ?\]

We can make the computation easier by using the distributive property. Since the factor 20 is multiplied by both of the other numbers, we can rewrite the expression as the product of 20 and the difference of the other two numbers.

\[(20 \times 18) - (20 \times 8) = 20 \times (18 - 8) = 20 \times 10 = 200\]

---

**Example**

Sarah wants to buy 4 sandwiches. Each sandwich costs $3.12, including tax. When she pays for the sandwiches, she will use a coupon for $2.00 off the total cost. How much will she pay for all of those items, with the coupon?

First, write an equation that could be used to solve the problem.

\[(\text{total cost of sandwiches}) - (\text{amount of coupon}) = (\text{amount Sarah pays})\]

\[(4 \times 3.12) - 2.00 = ?\]

Next, consider if any number properties can help you solve this problem. Instead of multiplying by a decimal to find the total cost, consider if the distributive property would help. Each sandwich costs $3.12. That can be rewritten as the sum $3.00 + $0.12. Use the distributive property and that sum to compute the total cost of the sandwiches.

\[(4 \times 3.12) = 4(3.00 + 0.12) = 4 \times 3 + 4 \times 0.12 = 12 + 0.48 = 12.48\]

Now, you can find the amount Sarah will pay.

\[(4 \times 3.12) - 2.00 = 12.48 - 2.00 = 10.48\]

**So, Sarah will pay $10.48 for the sandwiches if she uses the coupon.**

---

**Real-Life Example Completed**

**The Hot Dogs**
Here is the original problem once again. Reread it and underline any important information.

One day after diving, Cameron and some of the kids that he has met at the resort decide to eat some hotdogs on the beach. Cameron takes everyone’s order and heads to the hot dog stand. He figures that he will get a tray. Each of the kids has given him a few dollars and Cameron thinks that he has enough money to get everything.

When he gets to the stand, he checks the prices. Cameron needs to buy 9 hotdogs. The hotdogs are $1.50 for a plain dog plus $1.00 for cheese and sauce. Everyone wants cheese and sauce, so Cameron needs to buy nine hotdogs with cheese and sauce.

Given these numbers, how much will Cameron spend?

Cameron isn’t sure. He takes a napkin and asks for a pen so that he can figure it all out. He has $25.00. Does he have enough?

The Distributive Property will be very helpful to Cameron.

**Think about the Distributive Property. Let’s use it to write an expression to help Cameron.**

\[ 9(1.50 + 1.00) \]

Next, we can distribute the 9.

\[ 9(1.50) + 9(1.00) \]
\[ 9 \times 1.50 = 13.50 \]
\[ 9 \times 1 = 9 \]
\[ 13.50 + 9.00 \]
\[ = 22.50 \]

Cameron will spend $22.50 on the hotdogs. Out of $25.00, he will have $2.50 change.

---

**Vocabulary**

Here are the vocabulary words found in this lesson.

**The Distributive Property**  this property states that when a term is outside of the parentheses, that you multiply the term outside the parentheses with terms inside the parentheses. The property can be over addition or subtraction.
Equivalent equal. Equivalent expressions are equal expressions.

---

### Technology Integration

The following videos present explanations of the Distributive Property.

**KhanAcademyTheDistributive Property**

**The DistributiveProperty**

**James Sousa,Introduction toTheDistributive Property**

**James Sousa,Exampleof The DistributiveProperty**
Time to Practice

Directions: Use the Distributive Property to write an equivalent expression for each numerical expression.

1. $6(3 + 4)$
2. $5(4 + 1)$
3. $12(3 + 5)$
4. $6(7 + 8)$
5. $2(4 + 5)$
6. $3(5 - 2)$
7. $6(7 - 3)$
8. $5(4 - 2)$
9. $7(5 - 1)$
10. $6(9 - 3)$

Directions: Use the Distributive Property to write an equivalent expression for each variable expression.

11. $5(x + 3)$
12. $6(y - 2)$
13. $5(x + 9)$
14. $8(a + b)$
15. $7(x - y)$

Directions: Now take the problems in 1 - 15 and evaluate or simplify each expression.