Student Name ____________________________

Algebra II SOL Test Date – _____________________
Algebra II Formula Sheet
2009 Mathematics Standards of Learning

Geometric Formulas:

\[ A = \frac{1}{2}bh \]

\[ p = 4s \]

\[ p = 2l + 2w \]

\[ a^2 + b^2 = c^2 \]

Quadratic Formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \], where \( ax^2 + bx + c = 0 \) and \( a \neq 0 \)

Statistics Formula:

\[ z = \frac{x - \mu}{\sigma} \]

Permutations and Combinations Formulas:

\[ nPr = \frac{n!}{(n-r)!} \]

\[ nCr = \frac{n!}{r!(n-r)!} \]

Sequence and Series Formulas:

Given:

\[ a_n \] represents the value of \( n^{th} \) term

\[ S_n \] represents the sum of first \( n \) terms

\[ S_{\infty} \] represents the sum of an infinite geometric series

\[ r \] represents the common ratio

\[ d \] represents the common difference

Arithmetic

\[ a_n = a_1 + (n-1)d \]

\[ a_n = a_{n-1} + d \]

\[ S_n = \frac{n}{2} (a_1 + a_n) \]

\[ S_n = \frac{n}{2} [2a_1 + (n-1)d] \]

Geometric

\[ a_n = a_1 r^{n-1} \]

\[ a_n = a_{n-1} \cdot r \]

\[ S_n = \frac{a_1 (1 - r^n)}{1 - r}, r \neq 1 \]

\[ S_{\infty} = \frac{a_1}{1 - r}, |r| < 1 \]
Expressions and Operations
All.1  The student, given rational, radical, or polynomial expressions, will
a) add, subtract, multiply, divide, and simplify rational algebraic expressions;

<table>
<thead>
<tr>
<th>Hints and Notes</th>
<th>PRACTICE A.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>To add or subtract: Must have a common denominator</td>
<td></td>
</tr>
<tr>
<td>To multiply: Factor numerator, factor denominator, cancel common factors</td>
<td></td>
</tr>
<tr>
<td>To divide: Flip the fraction after the division sign and use multiplication rules</td>
<td></td>
</tr>
<tr>
<td>To simplify: factor numerator, factor denominator, cancel common factors – NO CHOPPING !!</td>
<td></td>
</tr>
<tr>
<td>Complex Fractions: Simplify numerator, simplify denominator, then divide</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1. Which is equivalent to $\frac{x^2 - 4}{x^2 - 4x + 4}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A $\frac{1}{x+1}$</td>
</tr>
<tr>
<td>B $\frac{x+2}{x-2}$</td>
</tr>
<tr>
<td>C $\frac{1}{4x}$</td>
</tr>
<tr>
<td>D $\frac{1}{x+4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Which is equivalent to $\frac{6a+12}{a} \cdot \frac{a^3}{a+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A $6a^2$</td>
</tr>
<tr>
<td>B $\frac{6}{a^2}$</td>
</tr>
<tr>
<td>C $\frac{6(a+2)}{a}$</td>
</tr>
<tr>
<td>D $\frac{6a^2 + 24a + 24}{a^3}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. Which is equivalent to $\frac{3x}{7} + \frac{5y}{14x}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A $\frac{8y}{21}$</td>
</tr>
<tr>
<td>B $\frac{x^2}{14}$</td>
</tr>
<tr>
<td>C $\frac{6x^2 + 5y}{14x}$</td>
</tr>
<tr>
<td>D $\frac{3x^2 + 5y}{14x}$</td>
</tr>
</tbody>
</table>
4. Which equivalent to \( \frac{x}{x+7} \)?

A \(-9\)
B \(\frac{x^2 - 9x}{(x+7)^2}\)
C \(\frac{x}{x-9}\)
D \(-\frac{1}{9}\)

5. Which is equivalent to \( \frac{1}{2} \cdot \left(\frac{x}{y} - \frac{4}{2+5}\right) \) ?

A \(\frac{x-4y}{5x+2y}\)
B \(\frac{y-4x}{2y+5x}\)
C \(\frac{x^2y^2}{(y-4x)(2y+5x)}\)
D \(2y^2 - 3xy - 20x^2\)

6. Which is equivalent to \(\frac{(a+b)^3}{18} \cdot \frac{2}{(a+b)^2}\) ?

A \(\frac{a+b}{9}\)
B \(\frac{(a+b)^2}{9}\)
C \(\frac{(a+b)^5}{36}\)
D \(18a + 9b\)

**All.1a SKILLS CHECKLIST:** I can...

- Add, subtract, multiply, and divide rational algebraic expressions.
- Simplify a rational algebraic expression with common monomial or binomial factors.
- Recognize a complex algebraic fraction, and simplify it as a quotient or product of simple algebraic fractions.
SOL AII.1
The student, given rational, radical, or polynomial expressions, will
b) add, subtract, multiply, divide, and simplify radical expressions containing
rational numbers and variables, and expressions containing rational exponents;
c) write radical expressions as expressions containing rational exponents and
vice versa.

HINTS AND NOTES

\[ b \sqrt{a^x} = \frac{a}{x^b} \]

Remember: “Denominator in Dip”

To add or subtract radicals:
Radicands must be the same. You may only add like radicals.
Always simplify your radical completely.
Pay attention to your root value. Everything is not a square root.

PRACTICE AII.1bc

1. Which expression is equivalent to \( \sqrt[3]{a^2} \)?
   - A \( \frac{3}{a^2} \)
   - B \( \frac{2}{a^3} \)
   - C \( \frac{1}{a} \)
   - D \( a^6 \)

2. Which is equivalent to \( \sqrt[3]{8x^6} \)?
   - A \( 2 \)
   - B \( 2x \)
   - C \( 2x^2 \)
   - D \( 2x^3 \)

3. Which is equivalent to \( \sqrt[3]{16} \)?
   - A \( 4 \)
   - B \( 8 \)
   - C \( 12 \)
   - D \( 32 \)
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Which is equivalent to ( \frac{1}{3} \frac{3}{2}a^2b^4 )?</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>( ab^3 )</td>
</tr>
<tr>
<td>B</td>
<td>( \sqrt[3]{ab} )</td>
</tr>
<tr>
<td>C</td>
<td>( \frac{3}{\sqrt[4]{a^2b^4}} )</td>
</tr>
<tr>
<td>D</td>
<td>( \frac{4}{a^2b^3} )</td>
</tr>
<tr>
<td>5. Which is equivalent to ( 2\sqrt{12} + 3\sqrt{3} )?</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>( \frac{16}{2} )</td>
</tr>
<tr>
<td>B</td>
<td>( 5\sqrt{15} )</td>
</tr>
<tr>
<td>C</td>
<td>( 7\sqrt{3} )</td>
</tr>
<tr>
<td>D</td>
<td>( 7\sqrt{6} )</td>
</tr>
<tr>
<td>6. What is the simplest form of ( \sqrt{72x^3} - 5x\sqrt{2x} )?</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>( x\sqrt{2x} )</td>
</tr>
<tr>
<td>B</td>
<td>( \sqrt{2x} )</td>
</tr>
<tr>
<td>C</td>
<td>( 2x\sqrt{x} )</td>
</tr>
<tr>
<td>D</td>
<td>( x^2\sqrt{2x} )</td>
</tr>
<tr>
<td>7. What is the value of ( \left( \frac{5}{2^5} \right)^{\frac{3}{2}} )?</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>( \frac{5}{2} )</td>
</tr>
<tr>
<td>B</td>
<td>( \frac{25}{4} )</td>
</tr>
<tr>
<td>C</td>
<td>( \frac{2}{5} )</td>
</tr>
<tr>
<td>D</td>
<td>( \frac{4}{25} )</td>
</tr>
</tbody>
</table>
8. Which is equivalent to the expression

\[ \sqrt[3]{16} + 3\sqrt[3]{54} - 2\sqrt[3]{81} \]

A  \( 11\sqrt[3]{2} - 6\sqrt[3]{3} \)
B  \( 11\sqrt[3]{2} - 2\sqrt[3]{3} \)
C  \( 2\sqrt[3]{2} \)
D  \( 5\sqrt[3]{2} - 6\sqrt[3]{3} \)

All.1b, c SKILLS CHECKLIST: I can…

- Simplify radical expressions containing positive rational numbers and variables.
- Convert from radical notation to exponential notation, and vice versa.
- Add and subtract radical expressions.
- Multiply and divide radical expressions not requiring rationalizing the denominators.
**SOL AII.1**
The student, given rational, radical, or polynomial expressions, will
d) factor polynomials completely

### HINTS and NOTES

Always look for a greatest common factor first  
\( xy + xw = x(y+w) \)

Look for patterns:
\[
\begin{align*}
    a^2 - b^2 &= (a + b)(a - b) \\
    a^2 + 2ab + b^2 &= (a+b)^2 \\
    a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\
    a^3 - b^3 &= (a-b)(a^2 + ab + b^2)
\end{align*}
\]

**square-multiply-square-opposite-plus**

***make sure you have that opposite sign in the second factor

You can multiply or “foil” your choices to work backwards, if you want to work backwards.

### PRACTICE AII.1d

1. Which is a factored form of \(9x^2 - 25\) ?
   - A \((3x-5)(3x+5)\)
   - B \((3x-5)^2\)
   - C \((3x+5)^2\)
   - D \((9x-25)^2\)

2. Which is a factor of \(16x^2 - 1\) ?
   - A \((x-1)\)
   - B \((4x+1)\)
   - C \((8x-1)\)
   - D \(4x\)

3. Which is a factor of \(x^2 - 2x - 15\) ?
   - A \((x-3)\)
   - B \((x-15)\)
   - C \((x+3)\)
   - D \((x+5)\)

4. Which is a factor of \(6a^2 + 5ab - 6b^2\) ?
   - A \((2a+3b)\)
   - B \((2a-3b)\)
   - C \((3a+2b)\)
   - D \((3a-3b)\)
5. Which is the factored form of \(8x^3 + 1\)?

- A \((2x - 1)(4x^2 + 2x + 1)\)
- B \((2x - 1)(4x^2 + 2x - 1)\)
- C \((2x + 1)(4x^2 - 2x + 1)\)
- D \((2x + 1)(4x^2 + 2x - 1)\)

6. Which is the factored form of \(1 - y^3\)?

- A \((1 - y)(1 + y + y^2)\)
- B \((1 - y)(1 + y - y^2)\)
- C \((1 + y)(1 - y - y^2)\)
- D \((1 + y)(1 + y + y^2)\)

7. Which represents the complete factorization of \(4x^2 - 14x - 8\)?

- A \(2(2x - 1)(x + 4)\)
- B \(2(2x + 4)(x - 1)\)
- C \(2(2x + 1)(x - 4)\)
- D \(2(2x - 1)(x - 4)\)

8. Given the area of a rectangle being \(2x^2 + 5x - 12\) Which of the following could represent the length of one side of the rectangle?

- A \(2x + 3\)
- B \(2x - 3\)
- C \(x - 4\)
- D \(x + 12\)

**All.1d SKILLS CHECKLIST: I can…**

- Factor polynomials by applying general patterns including difference of squares, sum and difference of cubes, and perfect square trinomials.

- Factor polynomials completely over the integers.

- Verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials.†
The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of $i$, and identify field properties that are valid for the complex numbers.

**HINTS and NOTES**

**TI-83 Calculator TIPS**

- Use your $i$ button on your calculator.
- Remember to include your parentheses. If $\frac{2+i}{3+i}$, then $\left(\frac{2+i}{3+i}\right)$

**Remember**

$i = \sqrt{-1}$ or just $i$

$i^2 = -1$

$i^3 = -1i$

$i^4 = 1$

This sequence repeats itself. In other words...

$i = i$  $i^5 = i$  $i^9 = i$

$i^2 = -1$  $i^6 = -1$  $i^{10} = -1$

$i^3 = -i$  $i^7 = -i$  $i^{11} = -i$

$i^4 = 1$  $i^8 = 1$  $i^{12} = 1$

Always change $i^2$ to -1 when working out by hand.

Any $\sqrt{-#} = i\sqrt{#}$

**PRACTICE A.2c**

1. Which expression is equivalent to $(6+2i)-(4+3i)$
   - A $2-i$
   - B $2+i$
   - C $2+5i$
   - D $10-i$

2. Which is equivalent to $(4-2i)(5+3i)$
   - A $26$
   - B $12$
   - C $14+2i$
   - D $26+2i$

3. Which is equivalent to $(4-3i)^2$?
   - A $25$
   - B $25-2i$
   - C $7$
   - D $7-24i$

4. Which is equivalent to $(3+2i)(2+5i)$?
   - A $-4+19i$
   - B $16+19i$
   - C $6+29i$
   - D $6-10i$
5. Which is equivalent to \( \frac{5+i}{1+3i} \)?

A \( \frac{4-8i}{5} \)

B \( \frac{4-7i}{5} \)

C \( \frac{1-7i}{5} \)

D \( \frac{-1-7i}{4} \)

6. Which is equivalent to \( \sqrt{3} \cdot \sqrt{-3} \)?

A \( 3i \)

B \( -3i \)

C \( 9 \)

D \( 9i \)

7. What number does \( i^{24} \) equal?

A \( i \)

B \(-1\)

C \(-i\)

D \(1\)
Put your answer in the box. These are open-ended questions. Work them out write your answer in the box (on the computer you would type your answer in the box being sure to put it in appropriate form, simplest fraction, decimal, etc.) For our purposes, you will write your answer in the box.

8. Simplify the following expression, \((4 + 2i)(4 - 2i)\). Type your answer in the box.

AII.3 SKILLS CHECKLIST: I can…

- Recognize that the square root of \(-1\) is represented as \(i\).
- Determine which field properties apply to the complex number system.
- Simplify radical expressions containing negative rational numbers and express in \(a+bi\) form.
- Simplify powers of \(i\).
- Add, subtract, and multiply complex numbers.
- Place the following sets of numbers in a hierarchy of subsets: complex, pure imaginary, real, rational, irrational, integers, whole, and natural.
- Write a real number in \(a+bi\) form.
- Write a pure imaginary number in \(a+bi\) form.
Equations and Inequalities
SOL All.4
The student will solve, algebraically and graphically,
  a) absolute value equations and inequalities
Graphing calculators will be used for solving and for confirming the algebraic solutions.

HINTS and NOTES
An absolute value equation or inequality makes TWO statements.

Shading of Graphs:
\[ |\text{absolute value}| \leq \text{number} \]
"AND" sentence (Less than or equal to)
Look for graph shaded between 2 numbers and closed circle

\[ |\text{absolute value}| \leq \text{number} \]
"AND" sentence (Less than only)
Look for graph shaded between 2 numbers and open circles

\[ |\text{absolute value}| \geq \text{number} \]
"OR" sentence (Greater than or equal to)
Look for graph shaded to the left and to the right, closed circles

\[ |\text{absolute value}| > \text{number} \]
"OR" sentence (Greater than only)
Look for graph shaded to the left and to the right, open circles

PRACTICE All.4

1. Which of the following represents the solution to \(|x| = 7| ?
   A x = 7
   B x = 0
   C x = -7
   D x = 7 or x = -7

2. Which inequality describes the solution set graphed

   A \(-3|4x - 1| + 7 < 2\)
   B \(-3|4x - 1| + 7 > 2\)
   C \(-2|6x - 1| + 5 < 3\)
   D \(-2|6x - 1| + 5 > 3\)

3. What is the solution to \(|2x - 3| - 1 < 3 | ?
   A \(\frac{-1}{2} < x < \frac{7}{2}\)
   B \(\frac{-7}{2} < x < \frac{7}{2}\)
   C \(x > \frac{-1}{2} \ or \ x < \frac{7}{2}\)
   D \(x = \frac{-1}{2} \ or \ x = \frac{7}{2}\)
4. Which inequality describes the solution set graphed?

![Graph with points at -3, 0, 1, 2, 3, 4, 5]

A $|3x-4| \geq 8$
B $|3x-4| < 8$
C $|2x-3| > 5$
D $|2x-3| \leq 5$

5. What is the solution set for

$|2x+5| = 7$

A $\{x | x = -6 \text{ or } x = 1\}$
B $\{x | x = -1 \text{ or } x = 1\}$
C $\{x | x = -6\}$
D $\{x | x = 1\}$

6. Which represents the graph of

$2|2x-1| > 10$

A

![Graph with points at -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

B

![Graph with points at -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

C

![Graph with points at -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]

D
7. Which of the following inequalities best represents

\[ |x - 1| < 3 \]

A. Solve absolute value equations and inequalities algebraically and graphically.

A.3 SKILLS CHECKLIST: I can...

☐ Solve absolute value equations and inequalities algebraically and graphically.
SOL AII.4
The student will solve, algebraically and graphically,
   b) quadratic equations over the set of complex numbers
Graphing calculators will be used for solving and for confirming the algebraic solutions.

HINTS and NOTES
Ways to solve a quadratic equation:
Put equation in \( ax^2 + bx + c = 0 \) form
1. Factor, set each factor equal to zero, find solutions
2. Use quadratic formula:
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]
3. Square Root both sides if a squared term is isolated on one side
4. Use your calculator:
   - Sketch in \( y = \)
   - Zoom 6
   - Look for zeros(x-intercepts)
5. Work backwards – “Plug it in !!!!” Substitute given answer choices into your calc and see what works

Know terminology: Solutions, zeros, roots, x-intercepts all mean the same thing

<table>
<thead>
<tr>
<th>PRACTICE 4b</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. What is the solution set for ( x^2 + 6x - 16 = 0 )?</strong></td>
</tr>
<tr>
<td>A ( {0,4} )</td>
</tr>
<tr>
<td>B ( {-8,2} )</td>
</tr>
<tr>
<td>C ( {-3,5} )</td>
</tr>
<tr>
<td>D ( {-2,8} )</td>
</tr>
<tr>
<td><strong>2. Which is the solution set for ( x^2 - 6x = 8 )?</strong></td>
</tr>
<tr>
<td>A ( {2 \pm 2i} )</td>
</tr>
<tr>
<td>B ( {2 \pm 2\sqrt{3}} )</td>
</tr>
<tr>
<td>C ( {4,2} )</td>
</tr>
<tr>
<td>D ( {-4,2} )</td>
</tr>
<tr>
<td><strong>3. Which is the solution set for</strong></td>
</tr>
<tr>
<td>( 2x^2 + 2x + 1 = 0 )</td>
</tr>
<tr>
<td>A ( {\pm \frac{1}{2}} )</td>
</tr>
<tr>
<td>B ( {-\frac{1}{2} \pm \frac{1}{2} i} )</td>
</tr>
<tr>
<td>C ( {-\frac{1}{2} \pm i} )</td>
</tr>
<tr>
<td>D ( {-1 \pm i} )</td>
</tr>
<tr>
<td><strong>4. What are the solutions to</strong></td>
</tr>
<tr>
<td>( (y+3)^2 - 81 = 0 )</td>
</tr>
<tr>
<td>A ( y = -12 \text{ or } y = -6 )</td>
</tr>
<tr>
<td>B ( y = -12 \text{ or } y = 6 )</td>
</tr>
<tr>
<td>C ( y = 12 \text{ or } y = -6 )</td>
</tr>
<tr>
<td>D ( y = 12 \text{ or } y = 6 )</td>
</tr>
</tbody>
</table>
5. What are the solutions to 
\[ x^2 - 3x - 4 = 0 \]
A \( x = 1 \) or \( x = -4 \)
B \( x = -1 \) or \( x = 4 \)
C \( x = \frac{3 \pm i\sqrt{7}}{2} \)
D \( x = \frac{3 \pm \sqrt{7}}{2} \)

6. What are the solutions to 
\[ x^2 - 3x - 4 = 0 \]
A \( x = 4i \) or \( x = -2 \)
B \( x = -4 \) or \( x = 2 \)
C \( x = 4 \) or \( x = 2i \)
D \( x = 4 \) or \( x = 2 \)

7. Which graph represents a quadratic equation with no real solutions?

A

B

C

D

All.4b SKILLS CHECKLIST: I can…

☐ Solve a quadratic equation over the set of complex numbers using an appropriate strategy.

☐ Calculate the discriminant of a quadratic equation to determine the number of real and complex solutions.
**SOL All.4c**

The student will solve, algebraically and graphically,
- c) equations containing rational algebraic expressions

Graphing calculators will be used for solving and for confirming the algebraic solutions.

**HINTS and NOTES**

To solve a rational equation:
- Eliminate your denominators
- Cross-multiply if possible
- Multiply both sides of equation by your Least Common Denominator

Plug it in! (See what answer satisfies your equation)

### Practice A. 4c

1. What is the solution to

   \[
   \frac{x}{2x+1} = \frac{4}{3}
   \]

   **A** \( x = \frac{-1}{5} \)
   **B** \( x = 5 \)
   **C** \( x = \frac{-4}{5} \)
   **D** \( x = \frac{-5}{4} \)

2. What value of \( q \) is the solution to the equation

   \[
   \frac{7q - 9}{6} = \frac{6q + 2}{4}
   \]

   **A** \( q = \frac{-11}{8} \)
   **B** \( q = -6 \)
   **C** \( q = \frac{31}{9} \)
   **D** \( q = 48 \)

3. What is the solution to

   \[
   \frac{3x^2 - 2}{x} = \frac{6x - 2}{x}
   \]

   **A** 6
   **B** 2
   **C** \( \sqrt{2} \)
   **D** 0
4. What is the value of y is the solution to the equation \( \frac{4y - 30}{3} + \frac{6y + 8}{2} = 9 \)

A \( y = \frac{28}{5} \)

B \( y = \frac{45}{13} \)

C \( y = \frac{8}{5} \)

D \( y = \frac{23}{24} \)
Put your answer in the box. These are open-ended questions. Work them out write your answer in the box (on the computer you would type your answer in the box being sure to put it in appropriate form, simplest fraction, decimal, etc.) For our purposes, you will write your answer in the box.

5. Solve the following equation for \( x \). Place your answer in the box.

\[
\frac{x}{x} + \frac{25}{x} = 10
\]
### SOL All.4d

The student will solve, algebraically and graphically, d) equations containing radical expressions
Graphing calculators will be used for solving and for confirming the algebraic solutions.

### HINTS and NOTES
To solve radical equation:
- If a squared equation, square root both sides
- If a cubed equation, cube root both sides

Plug it in !!! (see what answer satisfies your equation)

### PRACTICE All.4d

1. What is the solution set for
   \[ \sqrt{x - 4} = 5 \]
   - A \{21\}
   - B \{25\}
   - C \{29\}
   - D \{33\}

2. What is the solution set for
   \[ \frac{1}{4} \sqrt[9]{9 + x} = 1 \]
   - A \{-7, 7\}
   - B \{-5, 5\}
   - C \{7\}
   - D \{5\}

3. What is the solution set for
   \[ \sqrt{3y + 4} = 5 \]
   - A \{y | y = 3\}
   - B \{y | y = 1\}
   - C \{y | y = \frac{1}{3}\}
   - D \{y | y = \frac{1}{9}\}
4. What is the solution for \( \sqrt{x+16} = 3\sqrt{x} \)?

A \[ \{ x \mid x = \frac{1}{2} \} \]
B \[ \{ x \mid x = \frac{8}{5} \} \]
C \[ \{ x \mid x = 2 \} \]
D \[ \{ x \mid x = 5 \} \]

5. What is the solution to \( \sqrt[3]{x-3} + 3 = 5 \)?

A \( x = 2 \)
B \( x = 3 \)
C \( x = 7 \)
D \( x = 11 \)

6. The length, \( s \), (in feet) of the skid mark left by an automobile traveling at \( r \) miles per hour can be approximated by the relation \( r = 2\sqrt[5]{s} \). If a car is going 80 miles per hour when the brakes are applied, about how many feet long would the skid mark be?

A \( 320 \text{ ft} \)
B \( 410 \text{ ft} \)
C \( 640 \text{ ft} \)
D \( 1280 \text{ ft} \)

All.4d SKILLS CHECKLIST: I can...

☐ Solve an equation containing a radical expression algebraically and graphically.

☐ Verify possible solutions to an equation containing rational or radical expressions.
**SOL All.5**  
The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

### HINTS and NOTES

**Remember** your solution is your point or points of intersection.

If **given a graph**, look at your choices and approximate where the solutions are (Read your graph)

If **given equations**:
1. **Plug it in !!** Plug in choices to see which point satisfies both equations. You are working backwards and checking
   - Put equation one in \( y_1 \)
   - Put second equation in \( y_2 \)
   - Use Zoom 6, then adjust your window as needed
   - Then 2\(^{nd}\) Calc #5, put blinker on point, Enter, Enter, Enter
   - Be sure to check your answer(s).

***If your curves do not intersect at all – There is no solution to your system!***

### Practice All.5

1. **What is the solution set to the following system of equations?**
   \[
   \begin{align*}
   y + 2x &= 2 \\
   x^2 + 3y &= 22
   \end{align*}
   \]
   - A \( \{(-8,18) \text{ and } (2,-2)\} \)
   - B \( \{(-8,2) \text{ and } (18,-2)\} \)
   - C \( \{(-2,2) \text{ and } (18,-8)\} \)
   - D \( \{(8,-14) \text{ and } (-2,6)\} \)

2. **Which set of ordered pairs is the solution to the system of equations**
   \[
   \begin{align*}
   y &= x^2 - 2x - 1 \\
   y &= -x^2 + 4x - 1
   \end{align*}
   \]
   - A \( \{(0,3) \text{ and } (-1,2)\} \)
   - B \( \{(0,-1) \text{ and } (3,2)\} \)
   - C \( \{(0,-1) \text{ and } (6,23)\} \)
   - D \( \{(3,2) \text{ and } (6,-11)\} \)

3. **Which of the following is the solution to the system of equations below?**
   \[
   \begin{align*}
   2y &= x^2 - 6x - 9 \\
   2y &= -x^2 + 2x + 1
   \end{align*}
   \]
   - A \( \{(5,-7) \text{ and } (-1,-1)\} \)
   - B \( \{(1,1) \text{ and } (-5,23)\} \)
   - C \( \{(1,-7) \text{ and } (-5,23)\} \)
   - D \( \left\{\left(2, \frac{1}{2}\right)\right\} \)
4. Which is most likely the solution set for the system of equations?

A \( \{0,-1\} \) and \((-3,2)\)
B \( \{-1,2\} \) and \((-2,3)\)
C \( \{1,2\} \) and \(2,7\)
D \( \{-3,7,0\} \) and \(0,4,0\)

5. What is most likely the solution to the system?

A \( \{(1.5,2.5)\} \) and \(3,2\)
B \( \{-2.5,1.5\} \) and \(2,-3\)
C \( \{-2,-3\} \) and \(2.5,-1.5\)
D \( \{-3,2\} \) and \(1.5,-3\)

**All 5 Skills Checklist:** I can…

- Predict the number of solutions to a nonlinear system of two equations.
- Solve a linear-quadratic system of two equations algebraically and graphically.
- Solve a quadratic-quadratic system of two equations algebraically and graphically.
Functions and Statistics
### SOL AII.6

The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

### HINTS and NOTES

**Functions:**
1. **Absolute Value.** \( y = |x| \) Looks like a “V”. If positive, goes up. If negative goes down.
2. **Square Root** \( y = \sqrt{x} \) Looks like the path of a “Rocket” Has a definite starting point.
3. **Quadratic** \( y = x^2 \) “U” shape Can go up or down
4. **Exponential** \( y = 2^x \) “x in exponent”
5. **Logarithmic** \( y = \log x \) Inverse of exponential equation
6. **Rational Function** \( y = \frac{p(x)}{q(x)} \)
   Look for Horizontal and Vertical asymptotes
   (In calculator – \( p(x)+q(x) \) – use parentheses)
7. **Polynomial Function** \( y = ax^n + bx^{n-1} + cx^{n-2} + ... \) decreasing powers
   - \( x^2 \) equation: up/down or down/up Two movements (Parabola)
   - \( x^3 \) equation: up/down/up or down/up/down Three movements (cubic)
   - \( x^4 \) equation: up/down/up/down or down/up/down/up
     Four movements (“W”)

### PRACTICE AII.6

1. The graph below is an example of which type of function?

   ![Graph](image)

   - **A** Absolute Value
   - **B** Exponential
   - **C** Linear
   - **D** Quadratic

2. Which most likely represents the equation of the graph?

   ![Graph](image)

   - **A** \( y = \sqrt{4 - x} \)
   - **B** \( y = -\sqrt{4 - x} \)
   - **C** \( y = -\sqrt{4 + x} \)
   - **D** \( y = \sqrt{4 + x} \)
Transformations:
Remember horizontal shifts are the opposite of the sign
- \( y = |x - 2| \)
  Shifts 2 units to the right
- \( y = |x + 2| \)
  Shifts 2 units to the left

Vertical shift is at the end of your function and moves the same
- \( y = |x| + 2 \)
  Shift vertex up 2
- \( y = |x| - 2 \)
  Shift vertex down 2

Calculator: You can sketch every function in your calculator using \( Y= \) and zoom 6 or zoom 4

3. Which of the following is most likely the equation graphed

\[ y = (x + 2)^2 + 1 \]
\[ y = 5(x - 1)^2 - 2 \]
\[ y = (x - 2)^2 + 2 \]
\[ y = (x - 2)^2 - 1 \]

4. Which sketch could represent the graph of the function

\( f(x) = |x - a| \)

A
B
C
D
5. This is a portion of the graph of a polynomial function. If written in order of descending powers, which could be the first term of the polynomial?

A. $x^2$

B. $x^3$

C. $x^4$

D. $x^5$

6. Which could be the graph of $f(x) = ax^3 + bx^2 + cx + d$ if $a$, $b$, $c$, and $d$ are real numbers and $a < 0$?

A. 

B. 

C. 

D. 

**All.6 SKILLS CHECKLIST: I can…**

- Recognize graphs of parent functions.
- Given a transformation of a parent function, identify the graph of the transformed function.
- Given the equation and using a transformational approach, graph a function.
- Given the graph of a function, identify the parent function.
- Given the graph of a function, identify the transformations that map the preimage to the image in order to determine the equation of the image.
- Using a transformational approach, write the equation of a function given its graph.
SOL All.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include
a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) $x$- and $y$-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.
Graphing calculators will be used as a tool to assist in the investigation of functions.

HINTS and NOTES

Domain: $x$ values
Range: $y$ values
THINK: alphabetical order

To be a function: No two $x$'s are the same or it passes the vertical line test

Zero (root, solution, $x$-intercept) of a function: Where your function has a value of 0 (where your graph crosses the $x$-axis)

TI-83 Calculator:
• Put eqt. in $y=$
• Zoom 6 or 4
• $2^{nd}$ Calc #2 – zero
• Left Bound/Right Bound/Guess

Work backwards! Substitute the answers given to see which answer gives you a 0

To find value of a function: Plug the number into your expression for every $x$

To find composite functions: $f(g(x))$
Substitute the entire inner function into the outer function in the $x$ position

9.

This is a portion of the graph of a polynomial function. Apparently the function has a turning point at

A $(-1, -2)$
B $(-1, 3)$
C $(0, 1)$
D $\left(\frac{1}{4}, 0\right)$

10. Which describes the end behavior of the function $f(x) = x^3 - 4x^2 + 4x$ as $x$ approaches infinity?

A $y$ approaches $\infty$
B $y$ approaches 0
C $y$ approaches $-\infty$
D $y$ approaches 2
To find inverse of a function:

- $y = \text{switch } x \text{ and } y$
- Solve for $y$
- Inverse ($f^{-1}(x)$) will be what $y =$

On a graph:

- $x$ intercept – where your function crosses the $x$-axis
- $y$ intercept – where your function crosses the $y$-axis
- turning point – point where your function turns the corner and switches direction
- interval increasing – region on graph where $y$ values are increasing
- interval decreasing – region on the graph where $y$ values are decreasing

Asymptotes: Vertical & Horizontal

- Vertical: Look at denominator (whatever makes $x = 0$) $x = ?$
- Horizontal:
  - same power –
  - $y = \frac{\text{leading coefficient}}{\text{leading coefficient}}$
  - higher power below – no horizontal asymptote
  - higher power above – $y = 0$

End Behavior: Look at your graph and your question. Read carefully. It will ask you what $y$ value you are approaching as $x$ approaches a given value. If it is a rational function, you are always approaching your horizontal asymptote. Use your calculator!!

3.

This is a portion of the graph of a polynomial function. Apparently the function has a domain of

A $\{x \mid x \in \mathbb{R} \}$
B $\{x \mid x > 0\}$
C $\{y \mid y > 0\}$
D $\{x \mid -9 < x < 10\}$

4.

This is a portion of the graph of a polynomial function. This function apparently has the following types of roots

A 5 different real roots
B 4 different real roots
C 2 real roots and 2 imaginary roots
D no real roots

5. Which value is not a zero of $P(x) = x^3 + 3x^2 - x - 3$

A 1
B -1
C 3
D -3
6. This is portion of a polynomial function. Apparently the function has a double zero of

A  between -2 and -1  
B  between -2 and 1  
C  between 1 and 2  
D  between 3 and 4

7. The polynomial function \( f(x) = x^3 - 3x^2 + x + 1 \) has a zero between

A  -4 and -3  
B  -2 and -1  
C  -1 and 0  
D  3 and 4

8. If the domain of \( f(x) = 2x^2 - 3 \) is limited to \( \{-3,-1,1,3\} \), what is the range?

A  \{-21,-5,-1,15\}  
B  \{-21,15\}  
C  \{-1,15\}  
D  \{1,5,15,21\}

9. If \( f(x) = x^2 - 2x \) and \( g(x) = x - 3 \), which if the following expressions represents \( g(f(x)) \)?

A  \( x^3 - 5x^2 + 6x \)  
B  \( x^2 - 2x - 3 \)  
C  \( x^2 - 3x - 3 \)  
D  \( x^2 - 8x + 9 \)
10. If \( f(x) = 5x^2 - 7 \) what is \( f(-3) \) ?

A \(-52\)  
B \(-22\)  
C \(38\)  
D \(45\)

11. Which function represents the inverse of the function \( f(x) = x + 2 \)

A \( f^{-1}(x) = x - 2 \)  
B \( f^{-1}(x) = -x - 2 \)  
C \( f^{-1}(x) = -x + 2 \)  
D \( f^{-1}(x) = -(x + 2) \)
Highlight each correct answer. These questions give you choices. You must click on each correct answer and make sure you mark every answer that is correct. If you forget one, it will be incorrect. Circle each correct answer. More than one answer may be selected.

12. The graph of a polynomial function is graphed below. Highlight each expression that represents an increasing interval?
Click and Drag. These questions give you the choices for your answer or answers. You must click on each correct answer and drag it to the appropriate box. You must get all of them correct to get the answer correct. For our purposes, just write the correct answers in the boxes.

13. Click and drag the horizontal and vertical asymptotes for the function

\[ g(x) = \frac{x - 1}{x - 4} \]

Horizontal

Vertical

\[ x = 1 \quad y = 1 \quad x = 4 \quad y = 4 \quad x = -4 \quad y = -4 \]

All.7 SKILLS CHECKLIST: I can…

- Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically.
- Describe restricted/discontinuous domains and ranges.
- Given the graph of a function, identify intervals on which the function is increasing and decreasing.
- Find the equations of vertical and horizontal asymptotes of functions.
- Describe the end behavior of a function.
- Find the inverse of a function.
- Graph the inverse of a function as a reflection across the line \( y = x \).
- Investigate exponential and logarithmic functions, using the graphing calculator.
- Convert between logarithmic and exponential forms of an equation with bases consisting of natural numbers.
- Find the composition of two functions.
- Use composition of functions to verify two functions are inverses.
SOL All.8

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

**HINTS and NOTES**

**Solutions** of equation, **zeros** of a function, **roots** of the equation, and **x-intercepts** all refer to the **same thing**: Where is the value of my function equal to zero?

If zeros are $a, b, c$  Then factors are 

$$ (x-a)(x-b)(x-c) $$

**** Factors will contain opposite sign of roots

<table>
<thead>
<tr>
<th><strong>PRACTICE All.8</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Which is a zero of $f(x) = x^2 + x - 6$?</td>
</tr>
<tr>
<td>A -3</td>
</tr>
<tr>
<td>B -2</td>
</tr>
<tr>
<td>C 0</td>
</tr>
<tr>
<td>D 3</td>
</tr>
</tbody>
</table>

2. A polynomial function has a zero at $x = -4$. Which expression must be a factor of the polynomial?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A $x-4$</td>
<td>B $x+2$</td>
</tr>
<tr>
<td>C $x-2$</td>
<td>D $x+4$</td>
</tr>
</tbody>
</table>

3. Which of the following functions has x-intercepts of -2 and 1

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A $f(x) = x^2 - x - 2$</td>
<td>B $f(x) = x^2 + x - 2$</td>
</tr>
<tr>
<td>C $f(x) = x^2 - 2x + 1$</td>
<td>D $f(x) = 2x - 1$</td>
</tr>
</tbody>
</table>

4. Which of the following sets contains all the apparent zeroes for the function shown?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A {1}</td>
<td>B {-2, 0, 2}</td>
</tr>
<tr>
<td>C {-2, -1, 1}</td>
<td>D {-2, -1, 0, 1}</td>
</tr>
</tbody>
</table>
5. A section of the graph of the polynomial function with integral roots is shown. Which of the following sets most likely contain only elements that are factors of the polynomial?

\[ \{(x-2),(x-1.5)\} \]

\[ \{(x-2),(x-1),(x+1)\} \]

\[ \{(x+2),(x+1),(x-1)\} \]

\[ \{x,(x-2),(x-1),(x+1)\} \]

6. Which of the following sets contains all the apparent zeroes for the function?

\[ \{-2,2\} \]

\[ \{0\} \]

\[ \{-1,0,2\} \]

\[ \{-2,-1,0,1,5,2\} \]

**AII.8 SKILLS CHECKLIST: I can…**

- Describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
- Define a polynomial function, given its zeros.
- Determine a factored form of a polynomial expression from the x-intercepts of the graph of its corresponding function.
- For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function.
- Given a polynomial equation, determine the number of real solutions and nonreal solutions.
SOL AII.2
The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first $n$ terms, finding the $n^{th}$ term, and evaluating summation formulas. Notation will include $\sum_{i=1}^{n} X_i$ and $a_n$.

**HINTS and NOTES**

Formulas will be on your Formula Sheet, but not they are used for!!

Pay attention to whether you have an arithmetic (Adding) or a geometric (Multiplying) sequence

If you get stuck with the formulas — you can always just write all of your numbers out and find a term or add the numbers up to get the sum

**Arithmetic Term**

$$A_i = a_1 + (n-1)d$$

**Arithmetic Sum**

$$S_n = \frac{n(a_1 + a_n)}{2}$$

**Geometric Term**

$$A_i = a_1 \cdot r^{n-1}$$

**Geometric Sum**

$$S_n = a_1 \frac{(1-r^n)}{1-r}$$

**Infinite Geom. Sum**

$$S_\infty = \frac{a_1}{1-r}$$

**PRACTICE AII.2**

1. **What is the sum of the series defined by**

$$\sum_{n=0}^{4} (3 - 2n)$$

- A $-5$
- B $-3$
- C $-1$
- D $0$

2. **Two geometric means between 2 and 54 are**

- A $4$ and $12$
- B $6$ and $12$
- C $6$ and $18$
- D $12$ and $18$

3. **If $a_n = 2^{n-1}$, which number represents $a_4$?**

- A $15$
- B $8$
- C $7$
- D $6$

4. **What are two arithmetic means between 3 and 24?**

- A $8$ and $12$
- B $8$ and $16$
- C $9$ and $16$
- D $10$ and $17$
5. What is the value of \( \sum_{n=1}^{6} 2^n \)?

- A 62
- B 126
- C 128
- D 252

6. If \( a_n = 1 + \frac{1}{n} \), then what is \( a_9 \)?

- A \( \frac{11}{10} \)
- B \( \frac{10}{9} \)
- C \( \frac{9}{8} \)
- D \( \frac{3}{2} \)
Put your answer in the box. These are open-ended questions. Work them out write your answer in the box (on the computer you would type your answer in the box being sure to put it in appropriate form, simplest fraction, decimal, etc.) For our purposes, you will write your answer in the box.

7. Driving a piling into a harbor bottom, a pile driver sinks the piling 24 inches on the first stroke, 18 inches on the second stroke, and $13\frac{1}{2}$ inches on the third stroke. If the sequence is continued, how far will the piling be driven down on the 5th stroke?

All.2 SKILLS CHECKLIST: I can…

- Distinguish between a sequence and a series.
- Generalize patterns in a sequence using explicit and recursive formulas.
- Use and interpret the notations $\sum$, $n$, $n^{th}$ term, and $a_n$.
- Given the formula, find $a_n$ (the $n^{th}$ term) for an arithmetic or a geometric sequence.
- Given formulas, write the first $n$ terms and find the sum, $S_n$, of the first $n$ terms of an arithmetic or geometric series.
- Given the formula, find the sum of a convergent infinite series.
- Model real-world situations using sequences and series.
SOL All.9
The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

HINTS and NOTES
If given a graph, LOOK at the shape of your graph to determine if the Best Model would be Linear, Quadratic, Cubic, Exponential, or Logarithmic
- Linear – A line
- Quadratic – Parabola “U” shape
- Cubic – “Egyptian”
- Exponential – One side is increasing rapidly
- Logarithmic – Inverse of Exponential

If given a set of data, read carefully. If it states the type of model, then you are lucky and you use it.

If it states to determine the best model for the data, then do a rough sketch on your paper and you look to see which model is best.

TI-83 CALCULATOR
Remember to put x’s in List 1 and y’s in List 2. Clear anything in your lists.
- STAT-CALC
  - #4 Linear
  - #5 Quadratic
  - #6 Cubic
  - #9 Ln (Logarithmic)
  - #0 (10) Exponential
  - ENTER

PRACTICE A.8
1. In 1940, the life expectancy at birth in the general public was 62.9 years. By 1980 it had risen to 73.7 years. Assuming a linear relation, which is the best prediction of life expectancy in the year 2000?
   A 76.4
   B 79.1
   C 79.9
   D 84.5

2. Which is the best quadratic model for the data
   \{-1,2),(0,1),(2,5}\}
   A \(y = x^2\)
   B \(y = x^2 + 1\)
   C \(y = 2x^2 + x + 1\)
   D \(y = x^2 + 2x + 1\)

3. Which type of function would best model the data
   \[
   \begin{array}{c|cccccccc}
   x & 0 & 4 & 8 & 12 & 16 & 20 \\
   \hline
   y & 2 & 6 & 14 & 26 & 54 & 110 \\
   \end{array}
   \]
   A Quadratic
   B Cubic
   C Exponential
   D Logarithmic
4. What is the best model equation for the data

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>6</td>
<td>14</td>
<td>26</td>
<td>54</td>
<td>110</td>
</tr>
</tbody>
</table>

A $y = 2.21x + 1.20$
B $y = 2.21(1.20)^x$
C $y = 1.20(2.21)^x$
D $y = 2.21x^2 + 1.20$

5. What is your best prediction for $y$ when $x = 30$?

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>6</td>
<td>14</td>
<td>26</td>
<td>54</td>
<td>110</td>
</tr>
</tbody>
</table>

A 454
B 500
C 525
D 550

6. The table shows the number of students enrolled in the Algebra/Trig program at a high school the first 5 years the course was offered.

<table>
<thead>
<tr>
<th>Year (x)</th>
<th>No. of students (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
</tr>
<tr>
<td>3</td>
<td>84</td>
</tr>
<tr>
<td>4</td>
<td>97</td>
</tr>
<tr>
<td>5</td>
<td>108</td>
</tr>
</tbody>
</table>

Which of the following equations most closely describes the relationship between the number of students enrolled and the number of years the class has existed?

A $y = x + 13$
B $y = 13x + 43$
C $y = 10x + 23$
D $y = 13x - 43$
7. What is the best prediction for year #10?

<table>
<thead>
<tr>
<th>Year (x)</th>
<th>No. of students (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
</tr>
<tr>
<td>3</td>
<td>84</td>
</tr>
<tr>
<td>4</td>
<td>97</td>
</tr>
<tr>
<td>5</td>
<td>108</td>
</tr>
</tbody>
</table>

A 100  
B 150  
C 175  
D 225

All.9 SKILLS CHECKLIST: I can...

☐ Collect and analyze data.

☐ Investigate scatterplots to determine if patterns exist and then identify the patterns.

☐ Find an equation for the curve of best fit for data, using a graphing calculator. Models will include polynomial, exponential, and logarithmic functions.

☐ Make predictions, using data, scatterplots, or the equation of the curve of best fit.

☐ Given a set of data, determine the model that would best describe the data.
SOL All.10
The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

<table>
<thead>
<tr>
<th>HINTS AND NOTES</th>
<th>1. The time it takes to travel a given distance varies inversely as the average rate of travel. Averaging 42 miles per hour, it takes John 5 hours to drive to Pittsburgh. If it took him 4 hours and 20 minutes to reach Pittsburgh on his last trip, what was his average rate of travel?</th>
</tr>
</thead>
</table>
| **Direct Variation**- | A 36.4 mph  
B 46.7 mph  
C 48.5 mph  
D 49.4 mph |
| • \( \frac{y}{x} = k \) or \( y = kx \) is the form of a direct variation equation, where \( k \) is the constant of variation  
• y-intercept is always zero  
• The constant of the function is always the slope  
• Graphs of Direct Variation equations always cross through the origin  
• The constant can be negative or positive |
| **Inverse Variation**- | 2. The volume (V) of a sphere varies directly with the cube of its radius (r). If \( k \) is the constant of proportionality, which is the formula for this relationship? |
| • \( xy = k \) or \( y = \frac{k}{x} \) is the form of an inverse variation equation, where \( k \) is the constant  
• As input values increase, output values decrease and vice versa  
• Graphs of Inverse Variations are not linear |
| **Joint Variation**- | 3. Hooke’s law states that the force required to stretch a spring varies directly with the distance the spring is stretched. If a 10 pound force stretches a spring 2 inches, what force is required to stretch 5 inches? |
| • \( xyz = k \) or \( y = \frac{k}{xz} \) is the form of a joint variation equation, where \( k \) is the constant  
• You cannot graph a joint variation because it involves more than one variable |
| A 15 pounds  
B 20 pounds  
C 25 pounds  
D 30 pounds |
4. The amount of interest ($I$) owed on a loan varies directly with the length of time ($t$) of the loan. If $k$ is the constant of proportionality, which formula represents this relationship?

A $I = kt$
B $I = \frac{k}{t}$
C $t = kI$
D $t = \frac{k^2}{I}$

5. Boyle’s Law states that, for a fixed amount of gas, the volume of the gas at a constant temperature is inversely proportional to the pressure. If a certain gas occupies 9.84 liters at a pressure of 50 centimeters of mercury (cm Hg), what is the approximate pressure when the volume is increased to 12 liters?

A 39.8 cm Hg
B 41.0 cm Hg
C 43.2 cm Hg
D 45.1 cm Hg

6. In which of the following is $z$ directly proportional to $x$ and inversely proportional to the square of $y$?

A $z = k \frac{x^2}{y}$
B $z = kxy^2$
C $z = \frac{x}{y^2}k$
D $z = k \frac{y}{x}$

All.10 Skills Checklist: I can…

- Translate “$y$ varies jointly as $x$ and $z$” as $y = kxz$.
- Translate “$y$ is directly proportional to $x$” as $y = kx$.
- Translate “$y$ is inversely proportional to $x$” as $y = \frac{k}{x}$.
- Given a situation, determine the value of the constant of proportionality.
- Set up and solve problems, including real-world problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.
SOL AII.11
The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

HINTS and NOTES

Normal Distribution Chart

You must be able to draw your standard normal curve and know your percentages.

If your z-score is an integer, then you can use your normal curve for the percentage.

If your z-score is a decimal, then you will have to use your standard normal probability table.

Use your formula sheet!!!

PRACTICE A.10

1. At Thomas Nelson Community College, the Pre-Test for Mathematics has 200 points on the test. The mean is 120 and the standard deviation is 20. What is the approximate percentage of students which score below 100 points?

A 50%
B 34%
C 16%
D 7%

2. At Thomas Nelson Community College, the Pre-Test for Mathematics has 200 points on the test. The mean is 120 and the standard deviation is 20. In order to pass the Mathematics Pre-Test, a student must score 140 points. If 360 freshmen took the pre-test last fall, how many passed?

A 200
B 240
C 302
D 324

3. On his midterm exam, Jimmy scored 75 points, which was exactly 2 standard deviations above the mean. If the standard deviation for the test is 4, what is the mean for the test?

A 79
B 75
C 71
D 67

4. Susie's test grades in Algebra II for the third quarter are 100, 80, 60, 88, 92, and 90. How many scores are within one standard deviation of the mean?

A all 6 scores
B 5 scores
C 4 scores
D 3 scores
5. A normally distributed set of 968 values has a standard deviation of 11 and a mean of 108. Which is closest to the number of values expected to be above 125?

A 910  
B 989  
C 210  
D 59

Put your answer in the box. These are open-ended questions. Work them out write your answer in the box (on the computer you would type your answer in the box being sure to put it in appropriate form, simplest fraction, decimal, etc.) For our purposes, you will write your answer in the box.

5. Roanoke, Virginia had the following amounts of snowfall last January

4.2" 2.3" 6" 7.8" 10" 5.5" 12.5" 0.8"

Find the mean and the standard deviation. Type your answers in the appropriate box

Mean  Standard Deviation

6. A survey of 20 colleges found that seniors graduated with an average $12,000 in debt from student loans. The debt was normally distributed with a standard deviation of $3200. Find the probability that a senior graduated owing more than $16,000.

All.11 SKILLS CHECKLIST: I can…
- Identify the properties of a normal probability distribution.
- Describe how the standard deviation and the mean affect the graph of the normal distribution.
- Compare two sets of normally distributed data using a standard normal distribution and z-scores.
- Represent probability as area under the curve of a standard normal probability distribution.
- Use the graphing calculator or a standard normal probability table to determine probabilities or percentiles based on z-scores.
**HINTS and NOTES**

**Permutation:**
- Order matters
- Formula on formula sheet
- Examples are combinations on locks,
- A **combination lock** should be called a **permutation lock**!

**Combination:**
- Order does not matter
- Formula on formula sheet
- Examples are lotteries or the items in a fruit salad

**PRACTICE A.11**

1. If the digits can be repeated, how many 3-digit numbers can be formed using the digits 1, 2, 3, and 4?
   - A 24
   - B 48
   - C 64
   - D 256

2. Meredith has 14 girls on her softball team. She wants to have 2 co-captains. How many different choices does she have?
   - A \(14P_2\)
   - B 14!
   - C \(14C_2\)
   - D 14

3. If numbers and letters can be repeated, how many different 6-digit license plates can be made if the first two positions are letters and the last four are digits?
   - A 492,804
   - B 676,000
   - C 6,760,000
   - D 455,625
Put your answer in the box. These are open-ended questions. Work them out write your answer in the box (on the computer you would type your answer in the box being sure to put it in appropriate form, simplest fraction, decimal, etc.) For our purposes, you will write your answer in the box.

1. A committee of 3 teachers and 3 students is to be formed to judge a contest. If there are 7 students and 5 teachers to choose from, how many different committees could be formed? Type your answer in the box provided.

5. How many permutations can be formed using the letters in the word MATHEMATICS? Type your answer in the box provided.

All 12 Skills Checklist: I can…

☐ Compare and contrast permutations and combinations.
☐ Calculate the number of permutations of \( n \) objects taken \( r \) at a time.
☐ Calculate the number of combinations of \( n \) objects taken \( r \) at a time.
☐ Use permutations and combinations as counting techniques to solve real-world problems.
### Answer Key for Functions and Statistics

#### A.8

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Equation</td>
<td>Cost</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 8 Graphs highlighted
- 9 Boxes highlighted

#### A.9

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MLB</th>
<th>NFL</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>SD</td>
</tr>
<tr>
<td>V</td>
<td>V</td>
</tr>
<tr>
<td>MAD</td>
<td>MAD</td>
</tr>
</tbody>
</table>

#### A.10

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Boxes Highlighted</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### A.11

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 6

---

**Domain**

**Range**

---